Bilateral Bargaining with Externalities

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Cooperative Bargaining Theory

The Benefits

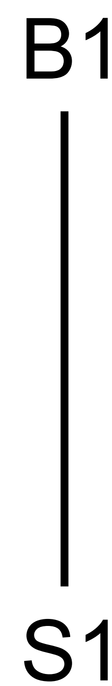
- Relates environmental characteristics to surplus division
- Easy to compute
  - E.g., Myerson-Shapley value is weighted sum of coalitional values

The Problems

- Presumption that coalitions operate to maximise surplus
  - Requires observable and verifiable actions
- Coalitional externalities are usually assumed away
  - If considered, impact on division only (Myerson)

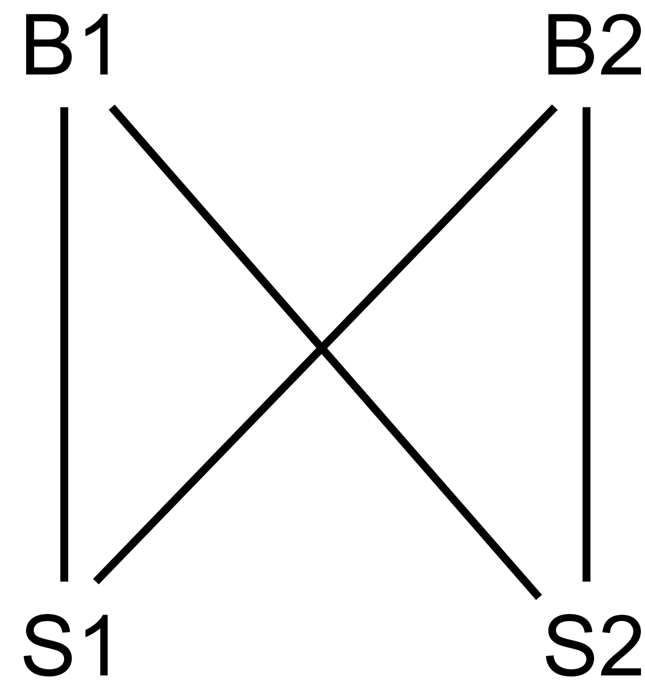
Non-Cooperative Bargaining Theory

- **Benefit**: Robust predictions in the bilateral case
  - Nash bargaining
  - Rubinstein and Binmore-Rubinstein-Wolinsky
- **Problem**: Bilateral case in isolation cannot deal with
  - externalities
  - coalitional formation



We need a theory that can deal with this …

- Competitive Externalities
  - Bs and Ss may be competing firms
  - Can't negotiate
- Bilateral Contracts:
  - Bs and Ss cannot necessarily observe supply terms of others
  - Connectedness does not necessarily imply surplus maximisation



… while being tractable and intuitive.

Our Approach

- Bilaterality
  - Assumes that there are no actions that can be observed beyond a negotiating pair
  - Potential for inefficient outcomes
- Non-cooperative bargaining
  - Does not presume surplus maximisation
  - Looks for an equilibrium set of agreements

Our Results

In a non-cooperative model of a sequence of bilateral negotiations …

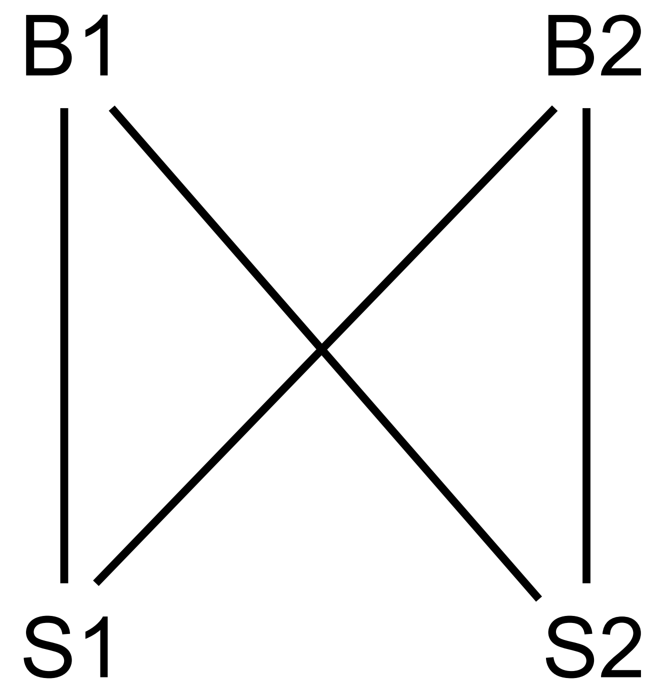
- There exists **a** Perfect Bayesian Equilibrium whereby
  - Coalitional surplus is generated by a Nash equilibrium outcome in pairwise surplus maximisation
  - Division is based on the weighted sum of coalitional surpluses
- We produce a cooperative division of a non-cooperative surplus
  - Strict generalisation of cooperative bargaining solutions
  - Collapses to known values as externalities are removed
  - Non-cooperative justification for cooperative outcomes

Buyer-Seller Networks

- The paper …
  - Considers a general structure with many actions (both individual and jointly observable), no restrictions on graph structure or on possible externalities
- This presentation
  - Focus on the 2x2 buyer-seller network where buyers are direct competitors
  - Illustrate game structure and results

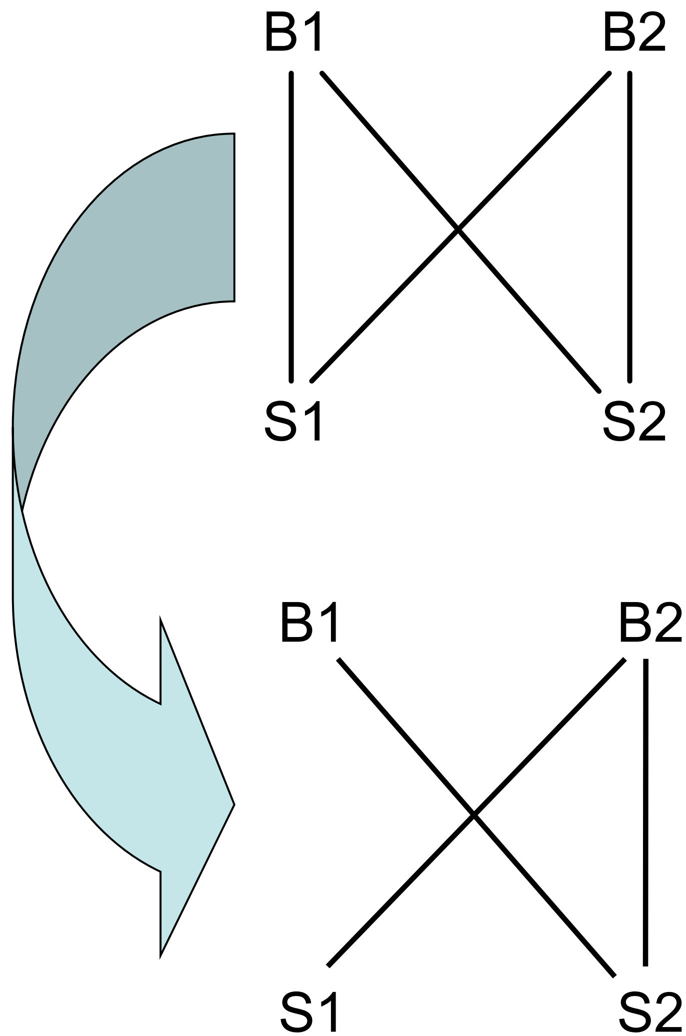
Some Notation

- Actions
  - x_{ij} is the input quantity purchased by Bi from Sj
  - t_{ij} is the transfer from Bi to Sj
  - **(A1)** Can only observe actions and transfers you are a party to (e.g., S2 and B2 cannot observe x_{11} or t_{11})
- Primitive Payoffs
  - Bi: $b(x_{i1}+x_{i2,} x_{-i1}+x_{-i2}) - t_{i1} - t_{i2}$
  - Sj: $t_{1j} + t_{2j} - c(x_{1j}+x_{2j})$
  - Usual concavity assumptions on $b(.)$ and $c(.)$



Network State

- Network
  - Bilateral links form a graph of relationships denoted by *K*
    - Initial state: *K* = (11,12,21,22)
  - If a pair suffer a breakdown (e.g., B1 and S1), the new network is created
    - New state: *K* = (12,21,22)
  - **(A2)** The network state (*K*) is publicly observed



Possible Contracts

- Bilaterality
  – As terms of other pairs are unobserved by at least one member of a pair, supply terms cannot be made contingent upon other supply contract terms
- Network Observability
  – As the network state is publicly observed supply terms can be made contingent on the network state
  – Example:
    - $x_{11}(11,12,21,22) = 3$ and $t_{11}(11,12,21,13) = 2$ and $x_{11}(11, 21,22) = 4$ and $t_{11}(11, 21,13) = 5$ and so on.

Extensive Form

- Fix an order of pairs (in this case 4)
  - Precise order will not matter for equilibrium we focus on
- Each pair negotiates in turn
  - Randomly select Bi or Sj
  - That agent, say Bi, makes an offer $\{x_{ij}(K), t_{ij}(K)\}$ for all possible K including Bi and Sj.
    - If Sj accepts, the offer is fixed and move to next pair
    - If Sj rejects,
      - With probability, 1-σ, negotiations end and bargaining recommences over the new network $K - ij$.
      - Otherwise negotiations continue with Sj making an offer to Bi.
- Binmore-Rubinstein-Wolinsky bilateral game embedded in a sequence of interrelated negotiations
  - Examine outcomes as σ goes to 1.

Beliefs

- Game of incomplete information
  - Need to impose some structure on out of equilibrium beliefs
  - Issue in vertical contracting (McAfee and Schwartz; Segal) in that one party knows what contracts have been signed with others and offer/acceptance choices may signal those outcomes
- Simple approach: impose *passive beliefs*
  - Let $\{\hat{x}_{ij}(K), \hat{t}_{ij}(K)\}_{\forall ij, K}$ be the set of equilibrium agreements
  - When *i* receives an offer from *j* of $x_{ij}(K) \neq \hat{x}_{ij}(K)$ or $t_{ij}(K) \neq \hat{t}_{ij}(K)$
  - *i* does not revise its beliefs about any other outcome of the game

Feasibility

- Focus on an equilibrium outcome that involves agreement in each bilateral negotiation
  - Maskin (2003): such an equilibrium may not exist due to free riding
  - Related to the existence of the core
- Bilateral Efficiency
  - A set of actions satisfied bilateral efficiency if for all *ij* in *K*,

$$\hat{x}_{ij}(K) \in \arg\max_{x_{ij}} b(x_{i1} + x_{i2}, \hat{x}_{-i1}(K) + \hat{x}_{-i2}(K)) - c(x_{1j} + \hat{x}_{2j}(K))$$

- (A3) Feasibility
  - Given any set of payoffs to all agents, any subset of them will be jointly better off with those payoffs than with the joint payoff they would receive if all existing links with those outside were severed; assuming that the resulting payoff satisfied bilateral efficiency.
  - With no component externalities, this is just a weak form of superadditivity

Equilibrium Outcomes: Actions

- Suppose that all agents hold passive beliefs. Then, as σ approaches 1, in any Perfect Bayesian equilibrium, each $x_{ij}(K)$ is bilaterally efficient (given K).
- Intuition
  - Negotiation order: 11,12,21,22 and suppose that 11 and 12 have agreed to the equilibrium actions
  - If 21 agree to the equilibrium action, 22 negotiate and as this is the last negotiation, it is equivalent to a BRW case – so they choose the bilaterally efficient outcome
  - If 21 agree to something else, B2 will know this but S2 wont
    - S2 will base offers and acceptances on assumption that 21 have agreed to the equilibrium outcome (given passive beliefs)
    - B2 will base offers and acceptances on the actual 21 agreement. Indeed, B2 will be able to offer (and have accepted) something different to the equilibrium outcome
  - Given this, will 21 agree to something else?
    - B2 will anticipate the changed outcome in negotiations with S2
    - Under passive beliefs, S1 will not anticipate this changed outcome (so its offers don't change)
    - B2 will make an offer based on:

$$x_{21} \in \arg\max_{x_{21}} b(x_{21} + x_{22}(x_{21}), \hat{x}_{11} + \hat{x}_{12}) - c(\hat{x}_{11} + x_{21}) - c(\hat{x}_{12} + x_{22}(x_{21}))$$

    - By the envelope theorem on x_{22}, this involves a bilaterally efficient choice of x_{21}.

Equilibrium Outcomes: Payoffs

- Result: As σ approaches 1, there exists a perfect Bayesian outcome where agents receive:

$$v_{B1} = \tfrac{1}{6}\left(3\left(\hat{b}(11,12,21,22) - \hat{c}(11,12,21,22)\right) + 2\hat{b}(1j,2j) - \hat{c}(1j,2j) - \hat{b}(i1,i2) + 2\hat{c}(i1,i2)\right)$$

$$v_{S1} = \tfrac{1}{6}\left(3\left(\hat{b}(11,12,21,22) - \hat{c}(11,12,21,22)\right) - 2\hat{b}(1j,2j) + \hat{c}(1j,2j) + \hat{b}(i1,i2) - 2\hat{c}(i1,i2)\right)$$

- This is each agent's Myerson-Shapley value over the bilaterally efficient surplus in each network.
- Sketch of proof:
  1. Impose passive beliefs: surplus generated is bilaterally efficient
  2. Demonstrate that entire surplus is allocated
  3. Demonstrate that payoffs satisfy fair allocation
  4. Myerson proof applies.

Remarks

- Stole and Zwiebel adopt a similar approach in proving their non-cooperative game yields a Shapley value
  - Make mistake: do not specify belief structure
- Our most general statement shows that the solution concept is a graph-restricted Myerson value in partition function space.
  - The symmetry in the buyer-seller network case masks some additional difficulties in the general case
  - There is some indeterminacy in the complete graph case
  - The cooperative game solution concept has never been stated before
  - Nor has it been related to component balance and fair allocation
  - So our proof does cooperative game theory before getting to the steps before

Ultimate Solution

$$\Upsilon_i(N,L) = \sum_{P \in P^N} \sum_{T \in P} (-1)^{p-1}(p-1)! \left[\frac{1}{|N|} - \sum_{\substack{i \notin T' \in P \\ T' \neq T}} \frac{1}{(p-1)(|N|-|T'|)} \right] \hat{v}(T, L^P)$$

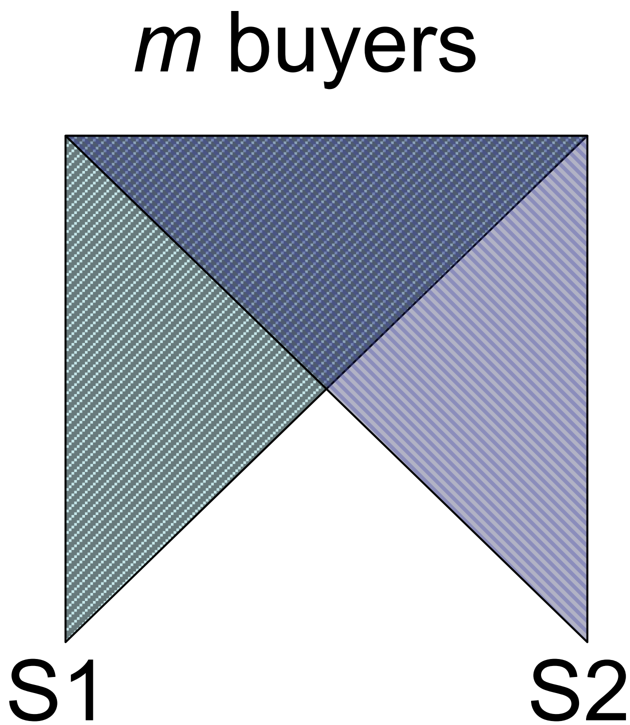
where:

- N is the set of agents
- P is a partition over the set of agents with cardinality p
- P^N is the set of all partitions of N
- L is the initial network (i.e., initial set of bilateral links)
- L^P is the initial network with links severed between partitions defined by P.

Additional Results

- (No component externalities) Suppose that primitive payoffs are independent of actions taken by agents not linked the agent
  - Obtain the Myerson value over a bilaterally efficient surplus.
- (No non-pecuniary externalities) Suppose that the primitive payoffs are independent of the actions the agent cannot observe
  - Obtain the Myerson value.
- If agreements are non-binding and subject to renegotiation, the results hold.

Computability



$$v_{S1} = \sum_{s=0}^{m} \binom{m}{s} \left(\sum_{i=0}^{x} \frac{(-1)^{s-i} \binom{s}{i}}{m-i+2} \right) \hat{v}(m-s,2)$$

$$+ \sum_{s=0}^{m} \sum_{h=0}^{m-s} \binom{m}{s} \binom{m-s}{h} \left(\sum_{i=0}^{m-s-h} \frac{(-2)^{m-x-h-i+1} \binom{m-s-h}{i}}{m-i+2} + \frac{(-1)^{m-s-h}}{m-h+1} \right) \hat{v}(s \mid h)$$

$\hat{v}(m-s,2)$ Bilaterally efficient surplus with *m*-*s* buyers supplied by both suppliers

$\hat{v}(s \mid h)$ Bilaterally efficient surplus if *s* buyers are supplied only by S1 and *h* are supplied only by S2

Applications

- Analysis of integration
  - Horizontal integration: amend Inderst & Wey (2002) to include competitive externalities
  - Vertical integration: de Fontenay & Gans (2003)
- Extending Stole-Zwiebel's wage bargaining model to more than one firm
- Exploring issues of network formation
  - Jackson & Wolinsky; Kranton & Minehart

Future Directions

- Can we generalise bargaining power?
  - Presumes equal breakdown probability and hence, equal allocations
- Can we generate efficiency?
  - What if renegotiation options were more fluid?
  - What information requirements can do this?
- What happens if feasibility is not satisfied?
  - Empty core but eventual equilibrium involving agreements amongst a smaller network of agents
  - Can compute this in applications