Intrafirm Bargaining with Heterogeneous Replacement Workers

by

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This paper provides a more general treatment of the version of Stole and Zwiebel's intrafirm bargaining model when workers can potentially be replaced. Specifically, we explore the robustness of their over-employment result to the introduction of heterogeneous replacement workers. Our main result demonstrates what type of heterogeneity can be used to nest both the no replacement and perfectly substitutable replacement models yielding over-employment and under-employment outcomes as extreme results. Consequently, we are able to identify the importance of firm-specific skills as a key driver of the extent of over- or under-employment by firms. *Journal of Economic Literature* Classification Numbers: C70, D23, J41, J64.

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The study of wage determination within firms has been bolstered in recent years by developments in the theory of multi-lateral bargaining. The most prominent example of this is the set of papers by Stole and Zwiebel (1996a, 1996b) (hereafter SZ). They argued that when a firm engages in one-on-one bargaining with a worker, their wage agreement is impacted upon by the outcomes of negotiations with other workers. Specifically, should negotiations with other workers break down, remaining workers will be in an improved bargaining position. Conversely, if the firm brought in more workers, existing workers would find their bargaining position diminished.[1]

SZ assumed that, at least in the short run, workers were *irreplaceable*. For this reason, a firm would have an incentive to ensure that its internal pool of workers was larger so as to depress the bargaining position of any one worker. In the long run, the firm would choose its set of workers such that the negotiated wage equalled workers' outside options.[2] In this sense, any individual bargaining power from workers is completely eliminated by the employment choice of the firm. This, of course, has a cost in that output is higher and the actual marginal product of labour is lower than it might otherwise be. However, from workers' perspectives, there are no rents available from employment (at the margin) and hence, no employment rationing by the firm. Indeed, with upward sloping labour supply curves there is greater equilibrium employment than would be the case in a strict neoclassical wage setting environment.

[1] Indeed, SZ were able to solve their model for explicit wage and profit solutions that were equivalent to the cooperative bargaining concept of the Shapley value. Smith (1999) provides a similar result based on a model where worker's appropriate some fraction of their marginal product. Interestingly, in that model competition for workers consolidates industries into larger firms who, on net, demand fewer workers in aggregate.

[2] As it is impossible to replace workers ex post, the firm chooses to expand its employment so as to reduce the hold-up power of any one worker. In fact, SZ demonstrate that a firm will hire workers up until the point at which an individual worker's negotiated wage equals their opportunity cost. The reason for this conclusion is based on the fact that, with intrafirm bargaining, the firm's payoff is an average of the profits it would receive as a neoclassical firm for each level of employment. This 'average' is maximised at the point where the 'marginal' function (in this case, neoclassical profits) cuts it from above. This implies that, at the profit maximising employment level, the firm's profits equal the profits they would receive if they had all the bargaining power (at that employment level). As a result, workers' wages equal their outside level.

In their analysis, it was understood that if any worker (or group of workers) who left the firm could be immediately replaced, wage determination would return to its neoclassical (or as SZ term it 'at will') drivers and there would be no over-employment or labour hoarding. While this seemed to suggest that, even with imperfect replaceability, firms would continue to over-employ to some degree, de Fontenay and Gans (2003) provided a small extension of the SZ model to demonstrate that this was not the case.

In that paper, we examined the possibility of ex post replacement from a *finite* pool of workers; such as might be drawn from other firms in the industry or from related jobs elsewhere in the economy. We found that, on the assumption that workers who left were immediately replaced, a closed form stable bargaining solution was possible whereby wages were a weighted average of the SZ wage and their reservation (or neoclassical) wage. Second, we found that, in general, this meant that the firm would choose to under- rather than over-hire relative to the neoclassical benchmark. This occurred because insiders had some hold-up power even with the threat of replacement, driving their actual wages up. As replacement was possible, the firm hired up to the point where marginal product equalled the bargaining wage. As this was above workers' reservation wages, under-hiring occurred. Importantly, it was only when there was *an infinite pool* of workers that employment levels returned to their neoclassical levels. Hence, this extension suggested that generic under-employment was likely.

Both the SZ model (with no replacement) and our variant (with replacement) lie at modelling extremes. In an important sense, SZ assume that outside workers are so different that insiders cannot be substituted for them while we assume that insiders and outsiders are perfect substitutes. This suggests that a bridge between the two results – allowing for a model more amenable to empirical analysis – would be found by considering the impact of worker heterogeneity. In so doing, we would be better able to understand the drivers of under- or over-employment at the firm level.

The paper proceeds as follows. In the next section, we briefly set up the intrafirm bargaining with replacement model. A first step in considering worker heterogeneity was to allow workers to have different reservation wages. This would naturally provide a division between eventual insiders and potential replacements on the basis of their wage (with the firm at first instance employing those with lower reservation wages). Section 2 demonstrates that this change does not overturn our earlier under-employment results. Section 3 then considers what happens when insiders and outsiders are imperfect substitutes in a productive sense. In that situation, the degree of substitutability does parameterise the overall level of employment moving from the extremes of over-employment (no substitutability) to under-employment (with perfect substitutability). This suggests that individual firm factors will play a critical role in any preferences for labour hoarding. Section 4 relates these findings to the issue of search costs incurred in finding replacement workers.[3] A final section concludes.

1. Model Set-Up

We begin by building on the SZ model in a simple way. Suppose that there is a single firm and that if it employs n workers, its revenue (or output) is $F(n)$. The pool of available workers is N, and each worker has a reservation wage of $\underline{w}$. This pool need not be of unemployed workers but of workers already employed by other firms

[3] Such models involving search costs and labour market frictions beginning with Mortenson and Pissarides (1994) are increasingly popular in macroeconomics. Smith (1999), Cahuc and Wasmer (2001) and Ebell and Haefke (2003) provide models embedding the standard SZ model with no

for whom the cost of matching and attracting them would be $\underline{w}$.[4] The wage outcome for an individual worker is $\tilde{w}_{N-n}(n)$ where the subscript indicates the number of 'outside' workers available and the number in parenthesis represents the number of 'inside' or employed workers.

The model has two stages:

STAGE 1: The firm chooses *n*, the number of workers it wishes to employ.

STAGE 2: The firm engages in one-on-one negotiations with each of these *n* workers on the basis of a split-the-difference (relative to their outside options) determination of wage payments. If negotiations with any of these break down, if there are available outside workers, the firm may choose to open up negotiations with these replacements.

In stage 2, we preserve the key assumption of SZ that wages are non-binding and can be renegotiated any time. While SZ provide an example of a non-cooperative game that might generate their bargaining solution,[5] for much of their results (and also in de Fontenay and Gans (2003)), they rely on an axiomatic definition of a stable outcome.

Definition 1 (Outcome). *An* ***outcome profile*** *is a collection of wages and profit levels,* $\{\tilde{w}_{N-n}(i), \tilde{\pi}(i)\}_{i=1}^{n}$*, one pair for each employee configuration, such that* $i\tilde{w}_{N-n}(i) + \tilde{\pi}(i) = F(i)$ *for all* $i \leq n$.

Definition 2 (Stability). *A* ***stable outcome profile*** *for n employees is an outcome profile such that for every workforce size,* $i \leq n$*, given split-the-difference renegotiations, no individual worker can improve upon their wage in a pairwise renegotiation with the firm, and the firm cannot improve profits renegotiating with any worker.*

Thus, a stable outcome will involve the firm and each worker being unable to improve upon their payoffs by triggering renegotiations. Importantly, this means that whenever there is a breakdown in negotiations between the firm and any one worker, even

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replacement; each obtaining over-employment results. Wolinsky (2000) provides a dynamic treatment but at the firm level.

[4] It is in the sense, that SZ and our model are partial equilibrium in nature, focussing purely on the choice of a firm taking labour market conditions as given. Smith (1999) and Cahuc and Wasmer (2001) endogenise this for the SZ no replacement model.

should the firm replace that worker, other workers may trigger renegotiations knowing that there is a smaller pool of replacements available.

To get a flavour of the outcomes under bargaining with replacement, re-call that the SZ bargaining wage (i.e., when there are no replacements) is:

$$\tilde{w}_0(n) = \tfrac{1}{n}F(n) - \tfrac{1}{n(n+1)}\sum_{i=0}^{n} F(i) + \tfrac{1}{2}\underline{w} \quad (1)$$

In de Fontenay and Gans (2003), we demonstrate that, so long as there is *immediate replacement* (i.e., the firm always replaces an insider if negotiations break down), then the bargained wage is:

$$\tilde{w}_{N-n}(n) = \left(\tfrac{n}{n+1}\right)^{N-n}\tilde{w}_0(n) + \left(1 - \left(\tfrac{n}{n+1}\right)^{N-n}\right)\underline{w}\,.$$

This is simply a linear combination of the SZ wage and the 'neoclassical' wage, $\underline{w}$ with $\lim_{N\to\infty}\tilde{w}_{N-n}(n) = \underline{w}$.

One convenient property of SZ is that comparable outcomes can be derived in a continuous version of their problem. We will rely on that case throughout this paper. Labour can be assumed divisible into infinitesimal units, each of which bargains pairwise with the firm. In this case, if the firm were to hire [0, n] workers, the negotiated wage is determined as follows (from SZ):

$$\tilde{w}_0(n) = \tfrac{1}{n}F(n) - \tfrac{1}{n^2}\int_0^n F(i)di + \tfrac{1}{2}\underline{w}\,.$$

When insiders can be replaced by a set, [n, N], of outsiders, under the immediate replacement assumption we can show that:[6]

[5] This non-cooperative game is applied only to their baseline model and, as demonstrated by de Fontenay and Gans (2004), the Shapley value outcome is only one of many possible Perfect Bayesian equilibria.

[6] The continuous case is found through calculus of variations. We suppose that breakdowns occur with groups of size *h* and take limits as *h* goes to 0. Suppose a quantity, *N*-*n*, is outside. Then in the event of a breakdown, a group of outsiders of size h would be brought in, and there would be (*N*-*n*-*h*) outsiders .Split-the-difference negotiations would imply $\left(F(n) - n\tilde{w}_{N-n}(n)\right) - \left(F(n) - n\tilde{w}_{N-n-h}(n)\right) = h\left(\tilde{w}_{N-n}(n) - \underline{w}\right)$ so that, by induction, $\tilde{w}_{N-n}(n) = \left(\tfrac{n}{n+h}\right)^{\frac{N-n}{h}}\tilde{w}_0(n) + \left(1 - \left(\tfrac{n}{n+h}\right)^{\frac{N-n}{h}}\right)\underline{w}$, which in the limit becomes the wage in equation (2). Alternatively, if we divide the split the difference negotiation equation by '*h*' and take the limit as *h*

$$\tilde{w}_{N-n}(n) = e^{-\frac{N-n}{n}}\tilde{w}_0(n) + (1 - e^{-\frac{N-n}{n}})\underline{w} \tag{2}$$

Once again, for any finite *N*, $\tilde{w}_{N-n}(n) > \underline{w}$ only equalling it in the limit as *N* gets very large.

Given this, we are now in a position to evaluate the firm's choice of insiders, *n*, from a potential pool of *N*. For this purpose, we will utilise the solution for the continuous case. The firm's profits are:

$$F(n) - \tilde{w}_{N-n}(n) = e^{-\frac{N-n}{n}}\tilde{\pi}(n) + \left(1 - e^{-\frac{N-n}{n}}\right)\pi(n).$$

Where $\tilde{\pi}(n) = \frac{1}{n}\int_0^n F(i)di - \frac{1}{2}\underline{w}$ are the SZ profits and $\pi(n) = F(n) - n\underline{w}$ are the 'neoclassical' profits. The firm will choose *n* to maximise this convex combination. This gives the first-order condition:

$$e^{-\frac{N-n}{n}}\frac{\partial\tilde{\pi}(n)}{\partial n} + \left(1 - e^{-\frac{N-n}{n}}\right)\frac{\partial\pi(n)}{\partial n} + \frac{N}{n^2}e^{-\frac{N-n}{n}}\left(\tilde{\pi}(n) - \pi(n)\right) = 0 \tag{3}$$

Let $\tilde{n}$ be the number of insiders that maximises SZ profits and n^* the number that maximises neoclassical profits. SZ show that $\tilde{n} > n^*$ as the firm always has an incentive to expand that number of workers from n^*.

In our case, what happens to the left-hand side of (3) if the firm chooses $n = n^*$, the neoclassical choice, when $n^* \leq N$? The first term of (3) is positive while the second term is zero. The third term is negative as SZ profits are the average of neoclassical profits, and are therefore less than neoclassical profits at their optimum. This suggests that, at the neoclassical optimum, the firm is torn between expanding the number of insiders and reducing the pool of outsiders. Note that the continuous version of the split-the-difference bargaining equation is (from SZ):

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goes to zero, we obtain the condition $-\frac{d(-n\tilde{w}_k(n))}{dk}\Big|_{k=N-n} = \tilde{w}_{N-n}(n) - \underline{w}$. The wage in (2) is the solution to this differential equation.

$$\frac{\partial \tilde{\pi}(n)}{\partial n} = \tilde{w}_0(n) - \underline{w} \tag{4}$$

Substituting (4) into the left-hand side of (3) we have, at $n = n^*$, that the left-hand side of (2) is:

$$-\left(\tfrac{N-n^*}{n^*}\right) e^{-\frac{N-n^*}{n^*}} \frac{\partial \tilde{\pi}(n^*)}{\partial n} < 0 \tag{5}$$

Hence, the firm will wish to reduce its employment below the neoclassical level. Basically, expanding employment reduces wages according to the SZ effect but the firm also reduces the outside pool. This outside pool depresses wages to the same extent but with a lower cost. Thus, with the possibility of immediate replacement, the firm chooses to under-employ rather than over-employ.

This suggests several things. First, note that as *N* gets very large, the profits become neoclassical so the firm chooses the neoclassical level of employment. Second, for finite *N*, at the chosen *n*, $\tilde{w}_{N-n}(n) > \underline{w}$. This stands in contrast to the SZ optimum where $\tilde{w}_0(\tilde{n}) = \underline{w}$. Consequently, insiders earn rents. Intuitively, note that the firm reduces the wage it pays, by hiring more insiders than the neoclassical firm. However, the marginal product of the extra insider necessarily is below this wage, above the neoclassical optimum. Without the possibility of replacement (as in SZ), this loss is mitigated by a reduction in infra-marginal wages. However, that same reduction can be achieved by the existence of a replacement worker. Therefore, the net result of moving an outsider inside the firm is a reduction in firm profits.

Moreover, note that because the negotiated wage is above $\underline{w}$ at n^*, $F'(n^*) < w_{N-n^*}(n^*)$. So the firm does not want to hire up to the neoclassical employment level. It rations employment as a result of the high negotiated wages.

One of the key assumptions made in the analysis thus far was the assumption of immediate replacement; that is, if negotiations broke down with an insider, the firm would choose to replace that insider immediately. For the case where the firm only wants a single worker, this assumption is innocuous as it would always be desirable to replace that worker. However, when the firm wants more workers, this assumption may not be reasonable. Without this assumption, we cannot obtain a closed form solution for wages and profits as we did in the previous section. Nonetheless, we can demonstrate that even in a more general specification, it remains true that when replacement is possible, the firm will under- rather than over-employ inside workers.

Proposition 1. *Under bargaining with-replacement the firm chooses* $n < n^*$, (*for* n^* *such that* $F'(n^*) = \underline{w}$), *and the wage in the stable outcome profile satisfies* $\tilde{w}_{N-n}(n) = F'(n) > \underline{w}$. *As* $N \to \infty$, $n = n^*$ *and* $\tilde{w}_{\infty - n^*}(n^*) = \underline{w}$.

The proof is in the appendix for the continuous case.[7] The intuition is very similar to that of the immediate replacement case. If outsiders are perfect substitutes for insiders you do not have to keep them inside the firm to benefit from their influence in depressing negotiated insider wages. Put simply, the firm does not lose anyone from the pool by bringing them inside the firm and paying them. Consequently, a worker is only brought inside if their marginal product exceeds the wage you expect to negotiate with them. Thus, at the optimum the bargained wage equals the marginal product. This wage always exceeds the neoclassical wage when the pool is finite, implying that there is a wage premium and also under-employment.

2. Replacements with Higher Reservation Wages

As noted in the introduction, our goal here is to explore the drivers of SZ over-employment and our under-employment result. The suggestion is that if replacement workers were different to those employed this might explain the level of employment of a firm. The paper explores heterogeneity amongst workers in two ways. In the next section, we consider differences in productivity between insiders and outsiders. In this section, we suppose that workers differ in their reservation wage; so that the firm faces an upward-sloping labour supply curve. One interpretation of this below is that the firm faces a friction in replacing workers from outside the firm. As we demonstrate, this type of heterogeneity still generates under-employment so long as there are some replacements with low enough reservation wages.

To examine reservation wage heterogeneity, for simplicity and consistency of exposition, we will consider the case of immediate replacement and a continuous labour force. Suppose that while all workers on [0, N] are equally productive, each worker i has a reservation wage of $\underline{w}(i)$. Critically, we assume that the firm and workers hold the belief that the reservation wage of each worker is independent of the firm's actions.

Without loss in generality, suppose that $\underline{w}(i)$ is non-decreasing in i. Thus, the firm considers employing the set of workers, [0, n] and replacing any subset, h, with the set of [n, $n + h$] workers. Taking the $\underline{w}(i)$'s as given, we can demonstrate[8] that the bargained wage to worker i is:

$$\tilde{w}_{N-n,i}(n) = e^{-\frac{N-n}{n}}\tilde{w}_{0,i}(n) + (1 - e^{-\frac{N-n}{n}})\tfrac{1}{2}\underline{w}(i) + \tfrac{1}{2n}\int_n^N e^{-\frac{i}{n}}\underline{w}(i)di \tag{6}$$

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[7] The proof for the discrete case is the appendix to de Fontenay and Gans (2003).

[8] Suppose that there are n insiders and a pool of outside workers, whose reservation wages range from $\underline{w}(k)$ to $\underline{w}(N)$; in this instance, k = N-n. Suppose breakdowns occur with groups of size h all of whom have the same reservation wage $\underline{w}(i)h$, and their negotiated wage is $\tilde{w}_{N-n-h,i}(n/h,h)h$. It is easy to show that the negotiated wages of any two workers with reservation wages $\underline{w}(i)h$ and $\underline{w}(j)h$ differ only in their final term, $\frac{1}{2}\underline{w}(i)h$ and $\frac{1}{2}\underline{w}(j)h$ respectively. The split-the-difference equation becomes (with some abuse of notation): $h\left(\tilde{w}_{N-n,i}(n) - \underline{w}(i)\right) \;=\; \tilde{\pi}_{N-n}(n) - \tilde{\pi}_{N-n-h}(n/h,h)$, so that:

$$h\left(\tilde{w}_{N-n.i}(n) - \overline{w}(i)\right) \;=\; \tilde{\pi}_{N-n}(n) - \left(\tilde{\pi}_{N-n-h}(n/h,h) + \tfrac{1}{2}\underline{w}(i)h - \tfrac{1}{2}\underline{w}(n+h)h\right).$$

Dividing by h and taking the limit as h goes to zero gives us the differential equation $\tilde{w}_{N-n,i}\left(0,n,n,N\right) - \frac{1}{2}\underline{w}(i) - \frac{1}{2}\underline{w}(n+h) = \frac{\partial\tilde{\pi}_{N-n}(n)}{n}$, which is solved by the wage in (6). A more detailed proof is available from the authors.

Thus, other insiders' reservation wages only impact on *i*'s wage through the SZ portion (first term) of (6).

Aggregating across the insiders, (6) leads to the profit equation:

$$\tilde{\pi}_{N-n}(n) = e^{-\frac{N-n}{n}}\tilde{\pi}_{0,i}(n) + (1-e^{-\frac{N-n}{n}})\pi(n) + (1-e^{-\frac{N-n}{n}})\tfrac{1}{2}\int_0^n \underline{w}(i)di - \tfrac{1}{2}\int_n^N e^{-\frac{i}{n}}\underline{w}(i)di \qquad (7)$$

where $\pi(n) = F(n) - \int_0^n \underline{w}(i)di$ is the neoclassical profits that would be received by a perfectly price discriminating firm. Let n^* be the employment choice of a neoclassical firm (i.e., so that $F'(n^*) = \underline{w}(n^*)$). Notice that these profits have a similar structure to profits with homogeneous workers, but with an additional term that reflects the heterogeneity of the pool of replacements relative to insiders. What this means is that as the firm chooses greater employment, not only is the pool of replacements reduced, its 'quality' in the firm's eyes is also lower. This means that the firm continues to have an incentive towards too few insiders relative to a neoclassical (i.e., perfectly price discriminating) outcome. This result is summarised in the following proposition.

Proposition 2. *Let n^* be the employment choice of a neoclassical firm (i.e., so that $F'(n^*) = \underline{w}(n^*)$). The firm's employment choice, n, is less than n^*.*

The proof is in Appendix. The intuition for the result is essentially the same as before.

Proposition 2 has important implications for a labour market characterised by competition. One interpretation of reservation wages is that these wages represent the alternative employment opportunities of workers – both inside and outside the industry. Suppose that workers in this industry can expect to earn some low reservation wage $\underline{w}$ in other industries. In addition, some of those workers are employed at other firms in the industry, and (if rational) they must be earning wages $\underline{w}(i) \geq \underline{w}$. The firm will hire and replace workers first from the set of unemployed workers (whose reservation wage is $\underline{w}$) and then, at need, from the workers employed at other firms.[9] If they do so taking the choices of other firms as given (e.g., as in Smith, 1999), then firms will choose to under-hire relative to what they would do if they were a neoclassical firm whose workers had reservation wages $\underline{w}$ and $\underline{w}(i) \geq \underline{w}$.

It is interesting to compare the labour market equilibrium for perfectly competitive (i.e. price-taking) neoclassical firms and for SZ bargaining firms. Suppose there are K neoclassical firms, N workers and the outside wage of those workers is $\underline{w}$. For any going wage w neoclassical firms would hire workers until $F'(n) = w$. Consequently the market equilibrium is to hire N/K workers if $F'(N/K) > \underline{w}$, and otherwise to hire $F'^{-1}(\underline{w})$ each. Now we consider the behaviour of a bargaining firm: suppose that every other firm were behaving as a neoclassical firm, the bargaining firm would under-hire relative to them. Therefore, there will necessarily be under-hiring when all firms engage in bargaining with workers as the reservation wages of replacements will be higher still. Thus, competition for workers will not eliminate the under-hiring result.

3. Replacements with Lower Productivity

A second source of heterogeneity between insiders and potential replacements concerns the issue of firm-specific skills. If the labour pool in question is the set of workers with firm- and industry-specific skills, and those skills are acquired by experience, then we would expect workers from other firms to be *imperfect* substitutes for one’s own workers in production. After a breakdown in negotiations with experienced insiders, a firm would be forced to replace them with workers from other firms, who are less productive from the firm’s perspective. This will have implications for the bargaining power of insiders.

[9] This is related to the literature on labour raiding (Lazear, 1986).

An interesting hybrid case is the case in which there is a fixed pool of available workers, but hiring them into the firm eventually confers firm-specific skills. One interesting point about this case is that it provides a smooth link between the SZ model and a model with a fixed pool. SZ effectively assume that potential replacements are never worth employing. We demonstrate that there is a smooth link between underemployment, in the case where outsiders are perfect substitutes, to over-employment as outsiders are worse and worse substitutes for insiders.

Suppose that the firm's production function is given by $F(n_1, n_2)$ where n_1 is the number of 'insiders' utilised in production and n_2 is the number of 'outsiders' utilised in production. An insider is a worker who was hired initially and has acquired a firm-specific skill or knowledge that cannot be acquired by outsiders who are hired only if an insider were to leave the firm. Thus, the timeline for our model is as follows: (1) the firm chooses a number of insiders, n_1 from a fixed pool of size N; (2) bargaining takes place between the firm and insiders with insiders potentially being replaced by an outsider from the 'replacement' pool $N_2 = N - n_1$; (3) production takes place with the firm utilising remaining insiders and any replacement outsiders, n_2.

Proposition 3. *Under bargaining with-replacement when outsiders have lower productivity, the firm chooses* $n_1 < n_1^*$ *(for* n_1^* *such that* $F'(n_1^*) = \underline{w}$*), whenever the difference in productivity is not too large and* $n_1 > n_1^*$*, otherwise. There exists a productivity differential such that* $n_1 = n_1^*$.

The proof is in the Appendix. In this case, there is over-hiring if the outsiders are poor substitutes for the insiders (as in SZ), and under-hiring if the outsiders are close substitutes. The limit case for poor substitutes is the SZ outcome, and the limit case for good substitutes is the perfect substitutes case as in Proposition 1.[10]

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[10] Note parenthetically that if the number of workers initially hired by the firm does not affect the size of the pool of reservation workers, then the firm over-hires until the bargained wage is equal to the reservation wage. The presence of the pool continues to depress the bargained wage, therefore the firm hires less than in the absence of a pool. By analogy, there is always over-hiring when the reserve pool

4. Search Costs

It is useful to illustrate Propositions 2 and 3 by considering what happens when there are search costs associated with hiring replacement workers. There are two ways search costs could impact upon intrafirm bargaining. First, each time there is a breakdown, the firm may have to incur a cost of *c*, in order to locate a suitable replacement. Following that, the firm negotiates with that replacement and all current workers. Second, search may involve a temporary cost of low productivity for a worker as finding them takes time. In this situation, for a time, insiders have superior productivity to the newcomers. Each of these 'search' costs has the quality that they are once-off rather than on-going.

The first type of search costs – an explicit upfront resource cost – is formally equivalent to locating replacements with a higher reservation wage. Therefore, suppose that all workers – insiders or outsiders – have a reservation wage of $\underline{w}$. Then, in split-the-difference negotiations, this will be taken into account as well as the cost of a replacement. For example, for a firm employing *n* workers from a pool of *N*, split-the-difference negotiations with any one worker will be determined by (when workers are discrete and there is immediate replacement):

$$\begin{aligned} &F(n) - n\tilde{w}_{N-n}(n) - \left(F(n) - n\tilde{w}_{N-n-1}(n) - c\right) = \tilde{w}_{N-n}(n) - \underline{w} \\ &\Rightarrow -(n+1)\tilde{w}_{N-n}(n) + n\tilde{w}_{N-n-1}(n) = -(\underline{w} + c) \end{aligned} \quad (8)$$

is infinitely large (falling to neoclassical levels as the productivity of reserve workers approaches that of insiders), as initial hiring does not affect the pool size.

The intuition for the over-hiring result is twofold. First, the two means of depressing hold-up power are now independent and therefore each is exercised to its optimal extent. Second, the firm is always better off with a larger pool of workers, and now the act of hiring increases the total pool the firm can draw on. Contrast this conclusion to a situation where workers have one of two different skill levels, but being high-skilled is no longer tied to initial presence in the firm: in that situation, the firm under-employs both types of workers.

While not especially illuminating from a theoretical standpoint, the case of a fixed outside pool may have relevant empirical applications. Consider the issue of training managers in-house, assuming that managers develop industry-specific and firm-specific skills. The number of managers trained by other firms may be independent of the firm's choice, and these other managers (and their industry-specific skills) can be drawn on if negotiations break down with the firm's in-house managers.

Therefore, Proposition 2 will apply and the firm will continue to under-hire relative to a neoclassical benchmark.

If, instead, the cost of search is that the position is not filled for some time, then this is similar to the loss of time while the new worker is developing skills. It translates to a productivity loss for the new worker. In that case, by Proposition 3, there will be under-hiring if these search costs are not too high (because search is relatively quick), and over-hiring if the search is very lengthy.

4. Conclusion

This paper has considered the SZ model in a context where replacement workers exist and those works are potentially distinct from one another and from insiders. We demonstrate that in this situation the under-employment result of de Fontenay and Gans (2003) arises where replacements are imperfect substitutes for insiders both in terms of productivity (the nature of their skills) and outside opportunity (with an upward sloping labour supply). In each case, we demonstrate how heterogeneous workers allow a nested model with SZ’s over-employment result at one extreme and under-employment at the other.

Our generalisation of the SZ model yields relatively clear predictions on over- and under-employment, based on the nature of workers’ specific skills. Human-capital specificity is decomposed into the initial size of the pool, the firm’s impact on the size of the pool, and the degree of substitutability between insiders and outsiders. These predictions have the potential to serve as hypotheses in future empirical research into employment of skilled workers at a firm level.

Appendix

Proof of Proposition 1

Let $n(N)$ be the number of workers that would maximise the firm's profits for a given total pool of available workers, N. Note that $0 \le n \le N$. That is, in the discrete case, for each N, the value $n(N)$ is the solution to:

$$\max_{n(N)} \pi(n(N), N)$$

$$\text{subject to } \pi(n(N), N) - \pi(n(N-1), N-1) = \tilde{w}_{N-n(N)}(n(N)) - \underline{w}$$

For the continuous case, suppose that labour is supplied in units of h.

$$\max_{n(N)} \pi(n(N), N)$$

$$\text{subject to } \pi(n(N), N) - \pi(n(N-h), N-h) = \left(\tilde{w}_{N-n(N)}(n(N)) - \underline{w}\right)h$$

By calculus of variations, the continuous case is found by taking the limit as $h \to 0$.

Stability implies that $\pi(n(N-h), N-h)$, profits when the pool is of size (N-h), are not influenced by the choice of $n(N)$ when the pool is of size N. This is because, after a breakdown, N falls to N-h and never returns to N implying that choices at N-h are not related to past choices of n. Thus, at N we can treat $\pi(n(N-h), N-h)$ as a constant.

Substituting terms for $\pi(.)$, we have:

$$\begin{aligned} &F(n) - n\tilde{w}_{N-n(N)}(n(N)) - K = h\left(\tilde{w}_{N-n(N)}(n(N)) - \underline{w}\right) \\ &\Rightarrow \tilde{w}_{N-n(N)}(n(N)) = \tfrac{1}{n+h}\left(F(n) - K + \underline{w}h\right) \\ &\Rightarrow \pi(n(N), N) = F(n) - \tfrac{n}{n+h}\tilde{w}_{N-n(N)}(n(N)) = \tfrac{1}{n+h}\left(F(n) - n(\underline{w}h - K)\right) \end{aligned}$$

where $K = \pi(n(N-h), N-h)$.

Any internal choice of n satisfies:

$$\frac{d\pi}{dn} = 0 = \frac{(n+h)\left(F'(n)h - \left(\underline{w}h - K\right)\right) - \left(F(n)h - n\left(\underline{w}h - K\right)\right)}{(n+h)^2}$$

This implies that $(n+h)F'(n)h - h\left(F(n) - \left(\underline{w}h - K\right)\right) = 0$. Thus, we have:

$$F'(n) = \tfrac{1}{n+h}\left(F(n) - K + \underline{w}h\right) = \tilde{w}_{N-n(N)}(n(N)) \qquad (9)$$

Therefore, whenever the boundary conditions $0 \le n \le N$ do not bind, the firm chooses n to equate the marginal product of an insider with the bargained wage that insider would be paid.

When N is small ($N \cong 0$), the firm wishes to hire all available workers; thus n is equal to N and wages are as in SZ, i.e., $\tilde{w}_0(n)$. As N rises beyond the point where $\tilde{w}_0(N) = F'(N)$, you do not hire all workers; but continue to satisfy (9).[11] Thus, $F'(n(N)) = \tilde{w}_{N-n(N)}(n(N)) > \underline{w}$ so that $n < n^*$.

Proof of Proposition 2

Let $\Phi(n) = \frac{1}{n}F(n) - \frac{1}{n^2}\int_0^n F(i)di$. Then,

$$\tilde{\pi}_{N-n}(n) = F(n) - ne^{-\frac{N-n}{n}}\Phi(n) - \tfrac{1}{2}\int_0^{N-n} e^{-\frac{j}{n}}\underline{w}(n+j)dj - \tfrac{1}{2}\int_0^n \underline{w}(j)dj .$$

Deriving with respect to n: and rearranging gives the first order condition:

$$(1-e^{-\frac{N-n}{n}})F'(n) + \left(-1-\tfrac{N}{n}+2\right)e^{-\frac{N-n}{n}}\Phi(n) \; - \tfrac{1}{2n^2}\int_0^{N-n}(n+j)e^{-\frac{j}{n}}\underline{w}(n+j)dj = 0 .$$

Setting $n = n^*$, the LHS becomes:

$$\begin{aligned}
&(1-e^{-\frac{N-n}{n}})\underline{w}(n) + \left(-1-\tfrac{N}{n}+2\right)e^{-\frac{N-n}{n}}\Phi(n) \; - \tfrac{1}{2n^2}\int_0^{N-n}(n+j)e^{-\frac{j}{n}}\underbrace{\underline{w}(n+j)}_{>\underline{w}(n)}dj \\
&< (1-e^{-\frac{N-n}{n}})\underline{w}(n) + \left(1-\tfrac{N}{n}\right)e^{-\frac{N-n}{n}}\Phi(n) \; + \tfrac{\underline{w}(n)}{2n}\int_0^{N-n} -\tfrac{n+j}{n}e^{-\frac{j}{n}}dj \\
&= (1-e^{-\frac{N-n}{n}})\underline{w}(n) + \left(1-\tfrac{N}{n}\right)e^{-\frac{N-n}{n}}\Phi(n) \; + \tfrac{\underline{w}(n)}{2n}\left((N+n)e^{-\frac{N-n}{n}} - 2n\right) \\
&= \left(\tfrac{N-n}{n}\right)e^{-\frac{N-n}{n}}\left(\tfrac{\underline{w}(n)}{2} - \Phi(n)\right)
\end{aligned} .$$

At n^*, $\frac{\underline{w}(n)}{2} - \Phi(n) = \frac{n}{2}\frac{\partial\tilde{w}_0(n)}{dn}$ which is negative (see SZ for proof). As such, it is worthwhile to set n less than n^*.

Proof of Proposition 3

Consider the situation once the firm has chosen the number of insiders N_1: from that point on, the number of good and bad workers (workers with and without firm-specific skills) is exogenously given by the number of breakdowns in bargaining that have occurred. The firm merely chooses how many of the bad workers N_2 to have inside the firm, $n_2(N_1, N_2)$.

Then for any value of N_1:

$$\pi(N_1, N_2) = F(N_1, n_2(N_1, N_2)) - n_2 w_2 - N_1 w_1$$

n_2 will be equal to zero at the initial N_1 chosen (if the firm had initially wanted positive amounts of the outsiders, it would bring them inside initially, increasing their productivity), until a point N_1^0. Below N_1^0, the value of n_2 will be interior until N_2=0.

[11] The larger the pool is, the lower is the bargained wage ceteris paribus, and therefore the lower $F'(n)$ must be to equal the bargained wage; thus as N increases the firm is gradually hiring in more workers, maintaining the equality. Differential equation solutions for $n(N)$ available from authors on request.

For interior n_2, we can prove the following (where F_i is the marginal product of workers of type i):

Lemma 1. *Suppose that* n_2 *is interior* ($0<n_2< N_2$)*, then* n_2 *will be chosen to equalise the wage and the marginal product of the worker (i.e.,* $F_2 = w_2$ *).*

PROOF: We use calculus of variations when labor is supplied in units of size h, as $h\to 0$, the firm solves: Max $\pi(n_1,n_2)$ s.t. SZ conditions:

$$\{F(N_1, n_2(N_1, N_2)) - N_1 w_1(N_1, N_2) - n_2(N_1, N_2) w_2(N_1, N_2)\}$$
$$- \begin{Bmatrix} F(N_1 - h, n_2(N_1 - h, N_2)) - (N_1 - h) w_1(N_1 - h, N_2) \\ -n_2(N_1 - h, N_2) w_2(N_1 - h, N_2) \end{Bmatrix} = h(w_1(N_1, N_2) - \underline{w})$$

and when $n_2 \neq 0$,

$$\{F(N_1, n_2(N_1, N_2)) - N_1 w_1(N_1, N_2) - n_2(N_1, N_2) w_2(N_1, N_2)\}$$
$$- \begin{Bmatrix} F(N_1, n_2(N_1, N_2 - h)) - N_1 w_1(N_1, N_2 - h) \\ -n_2(N_1, N_2 - h) w_2(N_1, N_2 - h) \end{Bmatrix} = h(w_2(N_1, N_2) - \underline{w})$$

Writing "K_1" for the sum of the second bracketed term in the top equation plus h times $\underline{w}$, and similarly for "K_2":

$$\Leftrightarrow \begin{cases} w_1(N_1, N_2) = \frac{1}{N_1 + h}\left(F(N_1, n_2) - n_2 w_2(N_1, N_2) + K_1\right) \\ w_2(N_1, N_2) = \frac{1}{n_2 + h}\left(F(N_1, n_2) - N_1 w_1(N_1, N_2) + K_2\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} w_1(N_1, N_2) = \frac{1}{N_1 + n_2 + h}\left(F + \frac{1}{h}(n_2 + h)K_1 - \frac{1}{h} n_2 K_2\right) \\ w_2(N_1, N_2) = \frac{1}{N_1 + n_2 + h}\left(F - \frac{1}{h} N_1 K_1 + \frac{1}{h}(N_1 + h)K_2\right) \end{cases} \quad \text{(A.1)}$$

$$\Rightarrow \pi = \frac{1}{N_1 + n_2 + h}\left(hF - N_1 K_1 - n_2 K_2\right)$$

We choose n_2 to maximise profits, taking decisions at (N_1-h, N_2) and (N_1, N_2-h) as given. When h is sufficiently small, the following approximation holds:

$$\Rightarrow \quad \frac{d\pi}{dn_2} = \frac{hF_2 - K_2}{N_1 + n_2 + h} + \frac{-hF + N_1 K_1 + n_2 K_2}{(N_1 + n_2 + h)^2} \approx 0$$

$$\Leftrightarrow \quad F_2 \approx \frac{F - \frac{1}{h} N_1 K_1 + \frac{1}{h}(N_1 + h)K_2}{N_1 + n_2 + h} = w_2$$

for any value of n_2 that is interior (not restricted by the boundary conditions $0\leq n_2 \leq N_2$). □

For any function of the function $n_2(N_1,N_2)$ at which n_2 is interior or zero ($n_2<N_2$), profits will be:

$$\pi(N_1, N_2) = F(N_1, n_2(N_1, N_2)) - n_2(N_1, N_2) F_2(N_1, n_2(N_1, N_2)) - N_1 w_1$$
$$= G(N_1, N_2) - N_1 w_1$$

We restrict attention to the case where the pool of outside workers N_2 is large enough that that an interior number of workers ($n_2 < N_2$) is demanded even at $N_1 = 0$. Then the above expression for profit will hold over the entire range of N_1. We can derive the expression for profits using the same logic as in the SZ continuous case:

$$\begin{aligned} \Rightarrow \pi(N_1, N_2) &= \int_0^{N_1} G(s, N_2)ds - \frac{\underline{w}N_1}{2} \\ &= \int_0^{N_1} \left[F(s, n_2(s, N_2)) - n_2(s, N_2)F_2(s, n_2(s, N_2))\right]ds - \frac{\underline{w}N_1}{2} \end{aligned} \qquad \text{(A.2)}$$

And pairwise bargaining implies that: $\frac{\partial \pi}{\partial N_1} = w_1(N_1, N_2) - \underline{w}$

Recall that there is a fixed pool of labour, N, with $N_1 + N_2 = N$, then the optimal choice of N_1 takes into account the effect on N_2:

$$\frac{d\pi}{dN_1} = \frac{\partial \pi}{\partial N_1} - \frac{\partial \pi}{\partial N_2} = 0 \quad \Leftrightarrow \quad w_1(N_1, N_2) = \underline{w} + \frac{\partial \pi}{\partial N_2}$$

Using this we will now demonstrate that:

(1) When bad workers have zero productivity (are not desirable) we are in the SZ situation, and $w_1 = \underline{w}$, because the partial of profits with respect to N_2 is zero.

(2) When the productivity of bad workers is in the limit equal to that of good workers, then in the limit $w_1 = F_1$. That is the result from the case where good and bad workers are identical (i.e. the firm does not want the workers inside the firm when workers cost more than they produce, because workers can be left outside the firm and drawn on in the case of breakdowns.). If the wage is equal to the marginal product, then the firm must be under-hiring, because at the neoclassical optimum $w_1 > \underline{w} = F_1$; so the firm would never hire up to the neoclassical optimum.

(3) Then by the Intermediate Value Theorem, for any smooth parameterisation of F_2 for which $F_2 = 0$ everywhere at one parameter value, and $F_2 = F_1$ at another, there is a parameter range over which under-hiring occurs, a range over which over-hiring occurs, and at least one value of the F_2 function for which exactly the neoclassical amount is hired in.

From (A.2) the partial derivative of profits with respect to N_2 is:

$$\frac{\partial \pi}{\partial N_2} = \frac{1}{N_1}\int_0^{N_1} \frac{dn_2}{dN_2}\frac{\partial\left[F - n_2F_2\right]}{\partial n_2}ds = \frac{1}{N_1}\int_0^{N_1} \frac{dn_2}{dN_2}\left[-n_2F_{22}\right]ds$$

Result **(1)** is immediate: If workers of type 2 are valueless, then the firm always chooses $n_2 = 0$, and this term is always zero. When this term is zero, the wage is equal to the SZ wage and is chosen as in SZ. □

To prove **(2)**, note next that:

$$\left.\frac{\partial\pi}{\partial N_2}\right|_{N_1=N_1^*} = \frac{1}{N_1}\int_0^{N_1}\frac{dn_2}{dN_2}\left[-n_2F_{22}\right]ds = \quad \frac{1}{N_1}\int_0^{N_1^0}\frac{dn_2}{dN_2}\left[-n_2F_{22}\right]ds = \frac{N_1^0}{N_1}\cdot\left.\frac{\partial\pi}{\partial N_2}\right|_{N_1=N_1^0}$$

where N_1^0 is the point above which $n_2 = 0$, because the integral is zero above N_1^0. Now below N_1^0, the value of n_2 is interior, and therefore $w_2 = F_2$. Bargaining implies that, at any value of N_1 at or below N_1^0:

$$\frac{\partial\pi}{\partial N_2} = w_2 - \underline{w} = F_2 - \underline{w}$$

Therefore,

$$w_1 - \underline{w} = \frac{N_1^0}{N_1}\cdot\left.\frac{\partial\pi}{\partial N_2}\right|_{N_1=N_1^*} = \frac{N_1^0}{N_1}\cdot\left[F_2(N_1^0) - \underline{w}\right] \qquad \text{(A.3)}$$

As bad workers come to resemble good workers in the limit, $F_2 \to F_1$; result **(2)** is therefore proven if $N_1^0 \to N_1$ as well.

We demonstrate that as the N_2s come to be identical to the N_1s, the wages of each (when both types are employed) are identical:

Recall equations (A.1): $$\begin{cases} w_1(N_1, N_2) = \dfrac{1}{N_1 + n_2 + h}\left\{F + \tfrac{1}{h}(n_2 + h)K_1 - \tfrac{1}{h}n_2K_2\right\} \\ w_2(N_1, N_2) = \dfrac{1}{N_1 + n_2 + h}\left\{F - \tfrac{1}{h}N_1K_1 + \tfrac{1}{h}(N_1 + h)K_2\right\} \end{cases}$$

where $K_1 = -\pi(N_1 - h, N_2) + \underline{w}h$ and $K_2 = -\pi(N_1, N_2 - h) + \underline{w}h$. As the two types of labour become identical: $\pi(N_1, N_2 - h) \to \pi(N_1 - h, N_2)$. Therefore $w_2 \to w_1$ at all values where both types of labour are employed, in particular in the limit as we approach N_1^0. Therefore at N_1^0:

$$w_2(N_1^0, N_2) - \underline{w} = F_2(N_1^0) - \underline{w} \quad = \quad w_1(N_1^0, N_2) - \underline{w} = \left.\frac{\partial\pi}{\partial N_1}\right|_{N_1^0} = \frac{F(N_1^0)}{N_1^0} - \frac{1}{(N_1^0)^2}\int_0^{N_1^0} G ds$$

Substituting the right-hand expression into (A.3) for $\left(F_2(N_1^0) - \underline{w}\right)$, we find that this is only satisfied when $N_1 = N_1^0$.

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