

## PART IV

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# **Marketing and Interdependent behaviour**

### Engendering Change

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When the decentralized decisions of individual agents in the economy lead to two or more equilibria that can be Pareto-ranked, the problem of *coordination failure* is possible. The economy could be trapped at a low efficiency equilibrium requiring the government, or some other external party, to intervene in an attempt to coordinate the actions of agents to reach a more efficient equilibrium.<sup>1</sup> Recently, such situations have been given renewed interest in formal economic theory (see Cooper and John 1988; Gans 1991). While this literature has been able to characterize the economic conditions that lead to coordination failure, and, hence, the need for intervention, virtually no attention has been given to what this need entails theoretically. It is the goal of this paper to make a start at addressing theoretically the particulars of facilitating transition between equilibria.

To restate: in its coordinating role, a government needs to convince individuals to change their behavior in order to facilitate an escape from a low equilibrium trap. A low efficiency equilibrium is a problem because it is stable, but unlike, for example, the equilibrium in a pure public goods problem, it is not globally stable. This means that if individuals can be persuaded to change their behavior by a sufficient amount, the conditions for a successful escape can be met and a virtuous cycle can begin. But changing individual behavior to provide the basis necessary for escape involves

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<sup>1</sup> The problem of coordination failure is related to the more familiar public goods problem. In those situations, the equilibrium outcome of interactions of agents may not coincide with the Pareto optimal resource allocation. In such a situation, it is the role of government to establish an equilibrium that yields the Pareto optimal allocation. In contrast, the role of government in the presence of coordination failure is to facilitate a movement from one equilibrium to another which is Pareto superior. Both types of problem share in common the notion that payoffs and decisions of self-interested agents are effected by the actions of other agents through nonmarket interaction.

changing their expectations and beliefs. These very beliefs have accommodated to the lower equilibrium leading individuals to take actions reinforcing those beliefs. Thus, the goal of government policy is to break the hold of *accommodation* by enough to form the basis for an escape.<sup>2</sup>

While escape, once begun, involves no additional role for the government, breaking the hold of accommodation of individuals involves substantial cost. These costs rise with the number of individuals who need to be targeted for change and with the degree to which they have to change their behavior. Thus, the optimal policy choice (i.e., the one that minimizes transition costs) often involves choosing between attacking on a wide front or storming the hill. The choice depends on how far one has to push on the front and how significant the hill is. And, in the context of game-theoretic models with multiple Pareto-rankable equilibria, it will be seen that the nature of accommodating beliefs can be the critical variable in the decision equation.

In order to motivate my analysis, in the next section, I describe the problem of discrimination in the labor market that possesses many of the characteristics of coordination failure problems. Section 2 will then discuss the dynamics of escape and what precisely needs to be done to generate upward momentum. Section 3 turns to consider the alternative (balanced and unbalanced) mechanisms for achieving individual change and some of the difficulties involved in the context of statistical discrimination. The economic characteristics that can drive the government's choice of mechanism are analyzed and discussed in Section 4. Section 5 concludes by addressing unresolved issues and directions for future research.

### **1 Specific Example: Discrimination in the Labor Market**

To analyze the policy choices facing governments in their coordinating role, it helps to have a specific context in mind. As a motivating context, I choose the situation of statistical discrimination in the labor market. Models of such situations have the quality that there can exist both discriminatory and non-discriminatory equilibria. Thus, the overall policy goal for the government would be to facilitate a transition away from discrimination. As such, the class of models I am examining are in the spirit of Arrow (1972a, 1972b, 1973) and Phelps (1972). According to these theories, discrimination is the result of self-fulfilling prophecies on the part of employers and potential workers, rather than being embedded in tastes per se (cf., Becker 1957). Looking to this type of statistical discrimination allows us to focus on

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<sup>2</sup> The terminology of "accommodation" and "escape" I borrow from Myrdal (1957) and Galbraith (1979).

some of the economic tradeoffs one would expect to confront a government in its coordinating role.

Although more complete analyses are present in the literature, in this paper I will only highlight the important *qualitative* properties of statistical discrimination models. As such, it is convenient here to focus attention on the discriminated against group which I assume is small relative to the labor market. This means that the labor market equilibrium for other groups need not be considered explicitly. In addition, for analytical and notational convenience, I assume that workers in the discriminated against group are drawn from a set  $\Lambda \equiv [0, 1]$ , a continuum.

Worker  $i$  receives a wage,  $w_i$ , and has marginal product  $q_i$ . However, this marginal product can be improved by investing in education or other personal factors, for example, “the habits of action and thought that favor good performance in skilled jobs, steadiness, punctuality, responsiveness, and initiative.” (Arrow 1972a: 105) Each worker chooses a single dimensional investment level,  $x_i$ , from a compact strategy space  $X_i \subset \mathfrak{R}$  (as in Kremer 1993). The payoff to worker  $i$  is assumed to be,  $\vartheta_i = \vartheta_i(w_i, x_i)$ , with  $\vartheta_{i1} > 0$  and  $\vartheta_{i2} < 0$ . So while additional investment is costly to workers, they do benefit if this results in a greater wage.

The insight of models of statistical discrimination is that if the marginal product of an individual worker is observed imperfectly, then it is possible that information concerning the productivity of other members of the group they belong to may influence the wage they receive.<sup>3</sup> Thus, the return a worker receives on their personal investment is dependent on the investments other workers from their group make. For example, often the labor market and final product market is assumed to be competitive with workers receiving their expected marginal product leading to the wage schedule,  $w_i = E[q_i | \mathfrak{T}_i]$ .<sup>4</sup> In such models, the expectation of a worker’s marginal product is based on the beliefs of employers who use information ( $\mathfrak{T}_i$ ), a composition of two types: (i) a test of the worker’s ability; and (ii) the knowledge of the societal group the worker belongs to and a point estimate of the general investment level of a worker in that group,  $\bar{x}$ . Here it is

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<sup>3</sup> As Arrow (1972a, 1972b) notes, the force of the imperfect information argument relies on employers making some specific investment in workers before they can observe their true productivity.

<sup>4</sup> See, for example, Arrow (1973), Lundberg and Startz (1983) and Kremer (1993). An alternative way of describing the problem of discrimination would be to assume that a worker’s observed ability affected their job placement. Thus, the returns to personal investment would take the form the quality of the position a worker received. Models along this line include Milgrom and Oster (1987), Coate and Loury (1993), and Athey et. al. (1993).

supposed that,  $\bar{x} = f(\{x_i\}_{i \in \Lambda})$ , where  $f$  is some aggregator function that is nondecreasing in each worker's investment. Its value influences the wage a worker from that group receives. As long as the test of a worker's ability is an imperfect indicator of marginal product, it can be shown that it will be optimal for employers to rely on their knowledge of  $\bar{x}$  in addition to the test (Phelps 1972; Kremer 1993). However, if the test provides some information to employers as to workers' individual productivities, their wages will be influenced, albeit imperfectly, by their own personal investment.

By undertaking personal investment, workers can be viewed as contributing to a public good,  $\bar{x}$ , that improves everyone's payoffs. This is because a higher  $\bar{x}$  results in higher wages for everyone in the group. What distinguishes this from a pure public good problem is that workers also receive a private benefit from their own personal investment, as a result of the use of an individual test by employers. Thus a worker's payoff becomes  $\vartheta_i = \vartheta_i(w(x_i, \bar{x}), x_i)$ , where  $w$  is nondecreasing in both its arguments. Under certain conditions, however, the wage function can have the quality that the marginal impact of individual personal investment on wages is nondecreasing in the aggregate investment,  $\bar{x}$  (Arrow 1974 and Kremer 1993). In such situations,

$$\frac{\frac{\partial \vartheta_i}{\partial w} \frac{\partial w}{\partial x_i}}{\frac{\partial \vartheta_i}{\partial x_i}} \text{ is nondecreasing in } \bar{x}.$$

This condition implies the definition of strategic complementarity as used by Milgrom and Shannon (1994).<sup>5</sup> For what follows I will assume this condition on payoffs.

Strategic complementarity present in worker's payoffs means that their best response correspondences are monotone nondecreasing in  $\bar{x}$  (Milgrom and Shannon 1994). In notation, let  $B_i(\bar{x}) \equiv \{x_i \in X \mid \pi_i(x_i, \bar{x}) \geq \pi_i(x'_i, \bar{x}), \forall x'_i \neq x_i\}$  denote the best response set

<sup>5</sup> In actuality, this game should probably be referred to as a game with aggregate strategic complementarities. This is because the strategies of other workers are embodied in the point estimate employer belief regarding the average level of worker education. This, in turn, is an aggregate of worker investments. Thus, it shares a common link with many applications of games with strategic complementarities (e.g., Cooper and John, 1988; Murphy, Shleifer and Vishny, 1989), but it is distinguished slightly from the game-theoretic literature on the subject (e.g., Milgrom and Roberts, 1990). In a technical appendix note reproduced here (Gans, 1994) I find the conditions under which games with aggregate strategic complementarities are a special case of games with strategic complementarities. The most important condition to note here is that the aggregator function be nondecreasing in each  $x_i$ .

for worker  $i$  when  $\bar{x}$  is produced. If  $B_i$  is single valued for worker  $i$ , this monotonicity is depicted in Figure 1. Thus, as other workers raise their personal investments, this raises  $\bar{x}$ , which in turn raises the optimal personal investment for an individual worker.

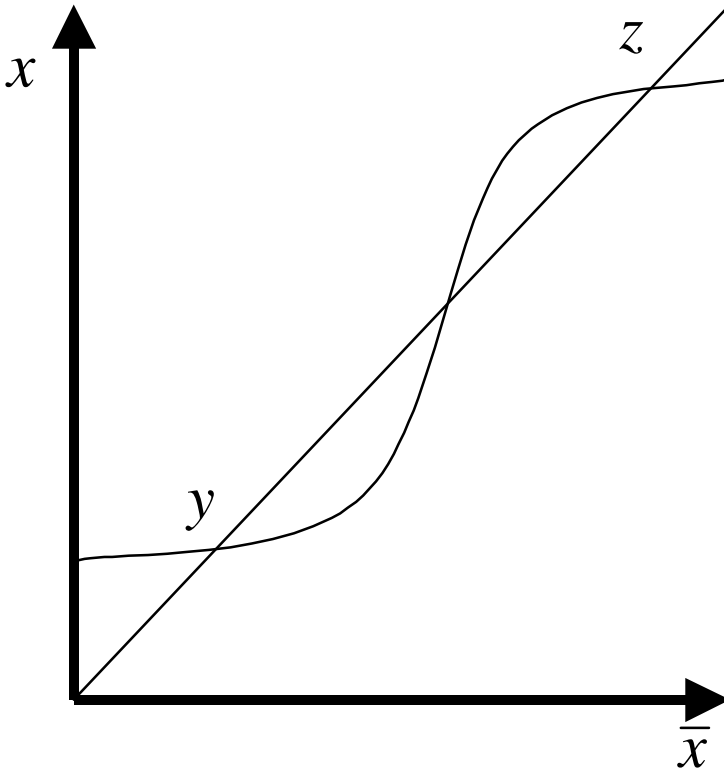


Figure 1. Multiple Equilibria

What are the possible equilibria of this type of game? To simplify matters, I will assume for the rest of the paper that all workers have the same underlying economic characteristics (although their observed characteristics may ultimately differ in equilibrium) and focus my attention on pure strategy symmetric Nash equilibria (SNE). As such, I will suppress the worker subscript in what follows.  $\hat{x}$  is, therefore, a SNE strategy profile for the discrimination game if (i)  $\hat{x} \in B(\bar{x}), \forall i$ ; and (ii)  $\bar{x} = f(\{\hat{x}\}; \theta)$ . Thus, in equilibrium, workers choose levels of investment that maximize their payoffs, and the aggregate level of education used by employers is consistent

with these maximal choices, capturing the notion of a self-fulfilling prophecy.

Strategic complementarities mean that there is the possibility of multiple equilibria (see Figure 1). There could be a low investment equilibrium with associated strategy profile,  $y$ , where workers invest little reinforcing low employer expectations regarding the level of productivity of a worker from the group. In addition, there could also be a high equilibrium, with strategy profile  $z > y$ , where workers invest high amounts in their productivity generating high employer expectations of the marginal product of individual workers from the group. And since  $\pi(x, \bar{x})$  is nondecreasing in  $\bar{x}$ , the highest equilibrium will always be preferred by workers (Milgrom and Roberts 1990, Theorem 6). In order to focus discussion on transition issues, in what follows I will assume that there are only two stable SNEs corresponding to strategy choices,  $y < z$ . Equilibria with higher outcomes are preferred by workers and since they are paid their marginal products, employers presumably prefer higher outcomes as well. Thus, it is a desirable policy goal for the government to facilitate a transition to the high equilibrium.

Much attention has been given to modeling how multiple equilibria can arise in a model of statistical discrimination<sup>6</sup> than what are the general tradeoffs a government faces in coordinating economic change. This is not to say that policy matters have not received attention. Indeed, the opposite is certainly true. Questions of differing policy goals and instruments under a taste-embedded versus a statistical discrimination setting have received much attention. In addition, throughout the social sciences there have been many empirical studies into the successes or otherwise of affirmative action policies and the like (e.g., Sowell 1983). However, in formal economics, there has been little synthesis and focus on the important considerations entering into government decision-making in the face of multiple equilibria

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<sup>6</sup> There have been numerous models within economics similar to the one sketched here. Gunnar Myrdal (1944) clearly had some notion that a positive feedback and complementarity between worker action and employer expectations lay at the heart of the discrimination problem. But it was not until Arrow (1972a, 1972b, 1973) and Phelps (1972) that the idea of modeling discrimination as a statistical phenomenon with multiple potential equilibria in the labor market was developed. Both of those theories emphasized imperfect information as lying at the heart of the problem, in that employers would find it advantageous to use group information in employer evaluations and that this could generate self-fulfilling prophecies and negative stereotypes. More recently, there has been a revival in such models focusing on discrimination within organizations that leave the observed wage schedule unaltered (Milgrom and Oster, 1987; Coate and Loury, 1993; Athey, Avery and Zemsky, 1994).

of the kind faced here. So while these models have, in the past, pointed out the need for change, there has been little discussion of what alternative mechanisms would be best for actually engendering change.

## 2 Dynamics of Escape

Gunnar Myrdal was the first economist to recognize the possibility that a virtuous circle could break the equilibrium of discrimination. Having outlined the feedbacks generating the problem, Myrdal continued,

White prejudice and low Negro standards thus mutually “cause” each other. If at a point of time things tend to remain about as they are, this means that the two forces balance each other: white prejudice and the consequent discrimination against the Negroes block their efforts to raise their low plane of living; this, on the other hand, forms part of the causation of the prejudice on the side of whites which leads them to discriminatory behavior.

Such a static “accommodation” is, however, entirely fortuitous and by no means a stable equilibrium position. If either of the two factors should change, this bound to bring a change in the other factor, too, and start a cumulative process of mutual interaction in which the change in one factor would continuously be supported by the reaction of the other factor and so on in a circular way. The whole system would move in the direction of the primary change, but much further. Even if the original push or pull were to cease after a time, both factors would be permanently changed, or the process of interacting changes would even continue without any neutralization in sight. (Myrdal 1957: 16-17)

Myrdal seems here to be implying that a virtuous circle could arise quite easily. However, as Swan (1962) points out, the notion of complementarity between the beliefs of employers and minority workers could also lead to vicious circles as well. It is the goal of this section to formalize and make precise such notions of circular and cumulative causation and in the process examine the requirements for a successful escape.

The hold of accommodation will be broken if the workers can be persuaded that it is worth their while to undertake personal investments in their productivity. In the situation described in Section II, the greater are the observed expectations of employers regarding the investment level of a given worker from the group, the larger will be an individual worker’s investment. Nonetheless, the danger is that if these observed employer beliefs are not high enough, the resulting investment will not justify even those beliefs and they will fall back to their low equilibrium levels. As it turns out, however, under certain dynamic assumptions, there does exist a critical level of aggregate employer beliefs such that, if those beliefs are generated, the process will not unravel and the hold of accommodation will be broken.

In showing how this is so, the first order of business is to make some assumptions about how beliefs evolve over time. It has already been assumed that employer beliefs about a worker's investment are some nondecreasing function of the current actual education investments of the group, i.e.,  $\bar{x} = f(\{x_i\}_{i \in \Lambda})$ . What is crucial, however, is that workers be persuaded to change their behavior. This will depend on their expectations regarding what the future level of employer beliefs will be. To see this, observe that at time  $t$ , each worker solves,  $\max_{x_{i,t} \in X} \pi(x_{i,t}, \bar{x}_t)$ . Here, each worker maximizes their payoff contingent upon their expectation regarding the aggregate employer beliefs in that period. The dynamic sequence of observation and action is depicted in Figure 2. For simplicity, it is assumed that when there are multiple best responses to a set of beliefs, then the highest strategy is chosen. Note, however, that is possible that sophisticated agents forming rational expectations could, even at strategies close to the high equilibrium, form expectations that drive them back to the low equilibrium. Thus, some restrictions on how workers use past observations of the aggregate to form their expectations are required to ensure that an escape, once begun, will continue.

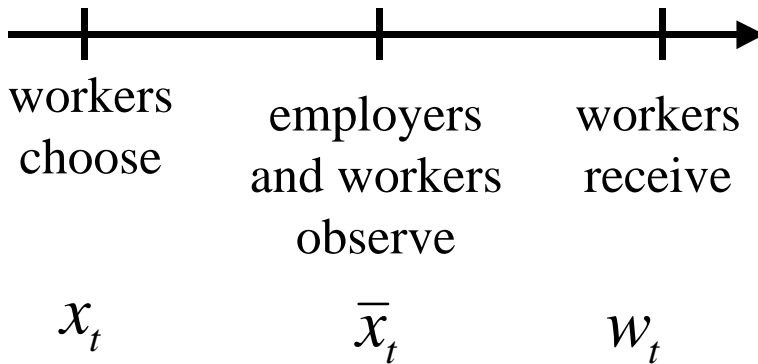


Figure 2. Time Line

Given this, how might workers adjust their expectations over time?  $\bar{x}_t$  is considered to be the state variable in the analysis that follows. Consider the following very weak definition of adaptive expectations.

**Definition.** Suppose that a worker's conditional probability density function of current employer beliefs is  $g(\bar{x}_t | \bar{x}_{t-1})$ . Expectations are adaptive if

$g(\bar{x}_t | \bar{x}_{t-1})$  satisfies the (generalized) monotone likelihood ratio property in  $\bar{x}_{t-1}$ , for any history  $\{\bar{x}_{t-2}, \bar{x}_{t-3}, \dots\}$ .<sup>7</sup>

The (generalized) monotone likelihood ratio property used in this way means that if the past aggregate estimate of worker education is higher, then the probability that this period's education aggregate exceeds any given level is greater. This merely says that if the previous period's personal investments for all workers improve, then so will their expectations regarding what this period's personal investment will be. If workers use only the immediate past observation of employer beliefs to form their expectations, i.e., setting  $\pi(x_{i,t}, \bar{x}_t) = \pi(x_{i,t}, \bar{x}_{t-1})$  and maximize that function, then the resulting dynamics are Marshallian or best reply dynamics. Thus, it can be seen that this often used adjustment dynamic is a special case of the definition of adaptive expectations given here.

**Theorem 1 (Momentum).** *Consider any game with aggregate strategic complementarities. Suppose that players adjust their strategies optimally (i.e., play highest best responses) using adaptive expectations. Then if  $x_t \geq x_{t-1}$  for some  $t$ ,  $x_s \leq x_{s+1} \leq \dots$  for all  $s > t$ . And if  $x_t \leq x_{t-1}$  for some  $t$ , then  $x_s \geq x_{s+1} \geq \dots$  for all  $s > t$ .*

This definition allows a variant of a theorem from Milgrom, Qian, and Roberts (1991) to be stated for multiperson decision contexts and games—their Momentum Theorem deals with adjustment processes in certain contracting problems.

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<sup>7</sup> A probability density function,  $g(x; \mu)$  satisfies the monotone likelihood ratio property (MLRP) in a parameter  $\mu$  if  $\frac{g(x; \mu)}{g(x; \mu')}$  is monotone nonincreasing in  $x$  for all  $\mu < \mu'$ . If  $G(x; \mu)$  is the probability that the random variable exceeds  $x$ , then the MLRP implies that  $G(x; \mu) \leq G(x; \mu')$ , for all  $x$ . This definition does not, however, allow for comparisons when changes in the parameter  $\mu$ , alter the support of the density. Suppose that  $x \in \mathfrak{R}$ , and let  $S(\mu) \equiv \{x | g(x; \mu) > 0\}$ . Then  $g(x; \mu)$  satisfies the (generalized) MLRP if given any  $x \in S(\mu)$  and  $y \in S(\mu')$ , (i)  $\min[x, y] \in S(\mu)$  and  $\max[x, y] \in S(\mu')$ ; (ii) for all  $\mu < \mu'$ ,  $\frac{g(x; \mu)}{g(x; \mu')}$  is monotone nonincreasing in  $x$  for all  $x \in S(\mu) \cap S(\mu')$ . The (generalized) MLRP is shown by Athey (1994) to be equivalent to Ormiston and Schlee's (1993) definition of MLR dominance. She also shows that the MLRP is a special case of the (generalized) MLRP. I allow for the more general definition here to allow for the possibility that agents' expectations put unit one mass on a single level of  $x$ , something essential in game theoretic contexts. This definition is also applied in Gans (1995).

**Proof.** *Since this is a game with strategic complementarities,  $p$  satisfies the single crossing property in  $(x_t; \bar{x}_t)$ . Moreover, by Theorem 5.1 of Athey (1994), since  $g(\bar{x}_t | \bar{x}_{t-1})$  satisfies the (generalized) MLRP in  $\bar{x}_{t-1}$ ,  $E[p]$  satisfies the single crossing property in  $(x_t; \bar{x}_{t-1})$ . Consequently, workers' best response correspondences are monotone nondecreasing in  $\bar{x}_{t-1}$  at each  $t$  (Milgrom and Shannon 1994). Suppose that  $x_t \geq x_{t-1}$ , we can conclude that  $\bar{x}_t \geq \bar{x}_{t-1}$ , since  $f$  is monotone in each worker's strategy. Then since each player's best response, at time  $t+1$ , is nondecreasing in  $\bar{x}_t$ ,  $x_{t+1} \geq x_t$ . The theorem then follows by induction. The proof of the second part is analogous to the first.*

Theorem 1 captures Myrdal's intuition that a virtuous cycle or escape, once begun, will have a momentum of its own. If ever there is a time such that workers wish to adjust their optimal investment upwards, that will feedback on itself to generate employer beliefs that continue to be justified by further upward movements in education for all workers. Note, however, that the theorem relies on expectations being adaptive. If expectations were not so then, in this general formulation, it could not be guaranteed that an upward cycle once begun would sustain its upward momentum.<sup>8</sup>

The above results shows that there exists some critical level of employer beliefs such that if, by intervention, a government can generate those beliefs, upward momentum will be generated and an escape from the trap of the low equilibrium will have been achieved. This critical level is defined, in the myopic best reply dynamic case, by,

$$(1) f^* = \arg \min_{\bar{x}} \{f(\{B(\bar{x})\}) \geq \bar{x}\}$$

If this critical level is generated, in the absence of any other shock, the state of play will not return to the discriminating equilibrium. What it does not say, however, is how far the escape will go. Under best reply dynamics, the outcome of play of the game will converge to the high static equilibrium. In other cases, the conditions of the theorem do not place enough restrictions to allow us to predict where play might converge to. For instance, the outcome of the path could be a Nash equilibrium or indeed, the state played could keep rising indefinitely, past the level of the high equilibrium.

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<sup>8</sup> Note also, that if agents were forward looking and maximized the present value of payoffs this period, i.e., solved  $\max_{\{x_{i,t}\}} \sum_{s \geq t} \delta^{s-t} \pi(x_{i,s}, \bar{x}_s)$ ,  $0 < \delta < 1$ , Theorem 1 would continue to hold. In addition, allowing for heterogeneous payoffs does not alter the momentum result if the player set is assumed to be finite. In that case, the result would state that if ever there was a time that a single worker adjusted optimal investment upwards, systemic momentum would follow.

Continued momentum, is, of course, desirable from a welfare point of view because the state,  $\bar{x}$ , enters positively into all worker's payoffs.

In order for the government to intervene and generate a level of the aggregate that escapes the basin of attraction of the low equilibrium, the adjustment process must be, in some sense, path dependent. That is, the adjustment path taken depends more on the history of past play than other factors (e.g., future expectations). Adaptive expectations have this property although convergence could be to an outcome other than the high static equilibrium. Best reply dynamics possess both an adaptive quality and predict convergence to some equilibrium<sup>9</sup>. Other dynamic adjustment processes that would also converge to some Nash equilibrium rely on more sophisticated expectations formation, but lose the path dependence that would prevent adjustment after intervention from returning play to the low equilibrium—e.g., rational expectations. If, however, one were to suppose that there were costs to the upward adjustment of strategies then it is possible that even under sophisticated expectations formation some separation of the basins of attraction of the high and low equilibria is possible.

There have been two general approaches to incorporating adjustment costs to analyze the transition between equilibria in games. In a series of papers, Matsuyama (1991, 1992) has analyzed dynamics in models with multiple equilibria where agents are only able to adjust their strategies intermittently. In addition, he assumes perfect foresight on the part of agents as to the future paths of states. What prevents dynamic paths from potentially returning to the low equilibrium is that, if the probability that an agent would be able to alter their strategy in any given period is low enough, the possible future paths for the state are constrained. Thus, if agents' discount rates are sufficiently high and if the initial condition of the system was above a critical state, then even if all other agents who were able to change used low strategies, it would still be optimal for any agent to play a high strategy. The intermittent adjustment process puts some lower bound on feasible paths for the state which in turn constrains even paths of rational expectation.

It would be beyond the scope of this paper to explore such adjustment dynamics in detail. In all other applications using this method to date, the strategy choice of agents is binary and the aggregate of concern is additively separable in each individual's strategy, for instance, some average. Nonetheless, if we were to view the government as intervening to change initial conditions, then the Matsuyama dynamic seems to imply that there

<sup>9</sup> Indeed, support for this type of adjustment behavior is part of the experimental literature (see Meyer et. al., 1992; Van Huyck, Cook and Battalio, 1992)

will exist some critical level of the aggregate such that if this is generated all future paths will converge to the high equilibrium.<sup>10</sup>

The other method by which one could incorporate more sophisticated expectations formation assumptions into adjustment dynamics and still guarantee convergence to the high equilibrium, is if the use of government policy *actually eliminated the low equilibrium*. Milgrom and Roberts (1991) have analyzed learning and adjustment processes in games. They define a class of learning processes that includes best reply dynamics and more sophisticated types of dynamics—such processes are consistent with adaptive learning. If educational changes were irreversible to some degree, then intervention may remove the low equilibrium. Then, as Milgrom and Roberts (1991) have shown, play will converge to the unique higher equilibrium regardless of the dynamic process assumed—whether sophisticated or merely adaptive. Gans (1994) demonstrates this process in more detail and the circumstances under which it might be relevant.

In summary then, a necessary element of any successful government policy to engender change is to ensure a sustained escape. Indeed, it could be argued that when considering policy in the face of coordination failure, it is critical that the details of the situation at hand imply some dynamic adjustment behavior that is path dependent for the government to intervene successfully. However, in situations where the dynamic permits a significant role for history, the goal of the government is to lift the aggregate of employer beliefs above some critical level,  $f^*$ . Beyond that level, transition will be completed by the adjustment behavior of individual workers. But generating that critical level depends on changing individual behavior already in the grip of accommodation. In order to avoid adding complicating conditions to the following analysis, I will suppose that the expectations of workers only depend on the immediate past level of aggregate employer beliefs. This removes any additional difficulties in generating the critical aggregate caused by long-standing low expectations since expectations will evolve in a similar manner to best reply dynamics. If this were not the case then, regardless of the dynamics adjustment behavior assumed, a government may have to intervene for longer periods of time in order to convince workers to improve their education in the face of greater employer beliefs. The other issues involved in achieving the critical aggregate are the subject of the next section.

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<sup>10</sup> For an excellent discussion of the roles of history and expectations along these lines see Krugman (1991)

### 3 Changing Individual Behavior

Given the discussion in the previous section, the transitional sub-goal of the government is clear: to generate the critical level of employer beliefs. One way to do this would be to intervene and change the beliefs of employers. But changing beliefs requires evidence and this is a scarce commodity in a world of self-fulfilling prophecies. Alternatively, one could legislate directly to avoid discrimination through an affirmative action policy. That is, one could move to equalize (observable) outcomes between groups. But recent theoretical discussions cast doubt on the effectiveness of outcome-based affirmative action policies because of enforcement difficulties (Lundberg 1991) and the potential for patronization of minority workers (Coate and Loury 1993).<sup>11</sup> Empirical doubts along similar lines are given by Sowell (1983) and Steele (1992).

Thus, the likely candidates for agents of change are the discriminated-against workers themselves. Since all workers contribute to generating employer beliefs, changing their incentives to invest in their productivity possesses the same qualities as getting individuals to contribute to the provision of a public good. The problem is that each worker perceives losses as a result of greater educational investment, although others making investments would enhance their own expected payoff. Given that we are currently at the low equilibrium, this means that to change an individual worker's behavior is bound to be costly. For instance, one possible mechanism for individual change would be to subsidize all workers for their expected *ex ante* losses on their educational return. But regardless of the actual mechanism, changing individual *i*'s investment from its low equilibrium value *y* to another value *x* entails a cost which I write as  $c_i(y, x)$ . This is the individual transition cost. So if the government were to push on a wide front in a balanced way, then, it needs to incur the individual transition costs of ensuring that all workers invest an amount  $x^*$ , where  $x^*$  satisfies  $f^* = f(\{x^*\}_{i \in \Lambda})$ . Thus, the total cost of this transition mechanism would be

$$(2) TC_b = \int_0^1 c_i(y, x^*) di = c(y, x^*)$$

This is the sum of all the individual transition costs.

But a balanced mechanism targeting all workers equally for change is not the only way a government could generate the critical level of employer beliefs. A myriad of unbalanced mechanisms could be imagined where only a subset of workers are targeted. For simplicity, suppose that when the

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<sup>11</sup> Schelling's (1978) discussion of the losses incurred by groups in transition also provide another set of arguments that diminish the desirability of outcome based affirmative action policies.

government targets some workers, it tries to induce them to change their strategy choices by the same amount. The balanced mechanism is a special case of this type of mechanism with all workers being targeted to change their strategy to  $x^*$ . Since  $f$  is nondecreasing, any mechanism that targets some subset of workers must necessarily induce those workers to change their strategy choice to some  $\tilde{x} > x^*$ . Thus, for any given target choice of strategy,  $\tilde{x}$ , some critical mass of workers would need to be targeted for change to  $\tilde{x}$  in order to generate the critical aggregate. The critical mass when workers are induced to invest  $\tilde{x}$ ,  $k^*(\tilde{x})$ , is determined by  $k^*(\tilde{x}) \in \{k \subseteq \Lambda | f^* = f(\{\{y\}_{i \notin k}, \{\tilde{x}\}_{i \in k})\}$ . All mechanisms targeting some subset of workers, i.e.,  $k^*(\tilde{x}) < 1$ , are called unbalanced transition mechanisms, with mechanisms with a lower  $k^*(\tilde{x})$  being described as more unbalanced.

Of course, inducing individual workers to change to some  $\tilde{x} > x^*$  is a harder task than getting them to change to  $x^*$ , and, therefore, more costly. Thus, I suppose, quite naturally, that  $c(y, x)$  is increasing in  $x$ . The total transition cost for any given unbalanced mechanism is,

$$(3) TC_u(\tilde{x}) = \int_0^{k^*(\tilde{x})} c_i(y, \tilde{x}) di = k^*(\tilde{x})c(y, \tilde{x})$$

Note that the definition of an unbalanced mechanism makes some implicit assumptions about the nature of the interaction between the government and employers. When the government intervenes to change the beliefs of employers by altering the investments of a group of workers, it is assumed that employers cannot distinguish between those workers who were targeted for change and those who were not. If they were able to make such a distinction then this might mean that employers separate their beliefs about targeted and non-targeted workers. In this case, the positive benefits of intervention will not spillover to the workers not targeted harming the possibility of an escape. Therefore, given the large set of workers, for analytical convenience this possibility is assumed away.

Therefore, as remarked upon earlier, the choice for the government is between moving a small amount on a wide front versus large pushes on limited fronts. To be sure, all mechanisms, balanced or unbalanced, will be sufficient to achieve a successful escape. Each breaks the hold of accommodation to the low equilibrium trap. But each mechanism may entail very different costs on the part of the government. As such, this will be its crucial dimension in the choice of alternative mechanisms. Having identified the important decision element for a government interested in engendering change, the rest of this paper will be directed towards identifying characteristics that will guide that choice.

#### 4 Characterizing the Optimal Policy Choice

In choosing the scope of its policy, i.e., the number of workers it targets, the government faces a tradeoff between the costs of targeting an additional worker and the costs of greater individual transition costs to change behavior by a greater amount. This tradeoff exists because, in order to ensure escape, any policy choice pair  $(\tilde{x}, k^*)$  is constrained to lie within the set,  $G \equiv \{ (x \in X, k \in \Lambda) | f(\{y\}_{i \notin k}, \{x\}_{i \in k}) \geq f^* \}$ . Thus, the policy choice of the government in its coordinating role is determined by the optimization problem,  $\min_{(x,k)} kc(y, x)$  subject to  $(x, k) \in G$ . In general, however, there is no guarantee that the functions and sets involved in this minimization problem are convex or even continuous. Thus, in order to say something more about what determines the optimal choice of the government, we need to adopt some parameterization of the relevant aggregate and of the transition costs.

Let me begin with the aggregate. Returning to the discrimination context, the relevant aggregate was employer beliefs regarding the average level of personal investment of the group of workers. Thus far, I have only assumed that employer beliefs could be represented by some function,  $\bar{x} = f(\{x_i\}_{i \in \Lambda})$ , that is nondecreasing in each worker's actual investment. It has not been necessary to ascribe a functional form for  $f$  for any of the results presented in the sections above. When it comes to analyzing the relative costs of balanced and unbalanced transition mechanisms, however, assuming specific functional forms becomes crucial.

Here I introduce a simple functional form as a description of how employers form their beliefs on the basis of observed educational levels of workers from the group. Employer beliefs regarding  $\bar{x}$  are determined by,

$$(4) \bar{x} = f(\{x_i\}) = \left( \int_0^1 x_i^a di \right)^{\frac{1}{a}}, a > 0$$

This is the commonly used CES aggregator (see Cornes 1993)<sup>12</sup> To see its properties observe that if  $a = 1$ , we have the ordinary mean of the investment levels,  $\int_0^1 x_i di$ . In this case by investing more, each worker contributes precisely that amount to the generation of improved employer beliefs. On the other hand, as  $a$  approaches 0, employer beliefs becomes a geometric average,  $\exp \int_0^1 \ln x_i di$ . Here, an investment by a worker contributes

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<sup>12</sup> This function form is a symmetric CES production function or the generalized mean of Hardy, Littlewood, and Polya (1952). It has been used in various forms in economics, recently, by Dixit and Stiglitz (1977), Romer (1987), and Cornes (1993).

a lower amount to employer beliefs if their investment is greater than that of other workers, but a greater amount if it is smaller than others' investments—so bad apples tend to spoil the bunch. Finally, observe that as  $\mathbf{a}$  approaches infinity,  $\bar{x}$  approaches the  $\max\{x_i\}$ . In this case, the worker with the greatest education level determines employer beliefs.

The parameter,  $\mathbf{a}$ , is, in this model, a measure of the *flexibility of employer beliefs*. If  $\mathbf{a}$  is high, employer beliefs adjust relatively easily to improvements in a number of workers' education levels. On the other hand, if  $\mathbf{a}$  is low, employer beliefs are relatively harder to change and improvements rely increasingly on the simultaneous improvement of many workers' education.

What considerations justify this parameterization of employer beliefs? One possible justification is that inflexibility (low  $\mathbf{a}$ ) describes a situation (i.e., type of labor market) where employers are more inherently prejudiced. This would result in a model that combines aspects of taste generated and statistical discrimination, although here inherent prejudice is embedded in the determinants of beliefs rather than in utility functions. Nonetheless, such an explanation might beg the question of why competition does not eradicate those employers holding such beliefs. This was the original motivation behind developing models of statistical discrimination in the first place (see Arrow 1972a).

That beliefs are rigid might originate in optimizing behavior. For example, as Kremer (1993) argues, employers may be using a production technology where the marginal product of workers of a given ability is raised by having co-workers of greater ability—the so-called “O-Ring” production function. In that case, employers would find it optimal to match workers of equal ability. The CES aggregate, therefore, represents what employers care about. Indeed, it is often a stated element of statistical discrimination that there are gains to the division of labor, i.e., complementarities, that preclude discriminated against workers from forming their own firms to compete with discriminating employers (see Arrow 1972b).<sup>13</sup>

Regardless of the explanation underlying it, with this specification for employer beliefs, one can see how flexibility,  $\mathbf{a}$ , will be critical in

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<sup>13</sup> An alternative explanation, based on an observation of Meg Meyer, is that the information employers receive about the group's aggregate ability comes from information sources that are unreliable and that alternative information sources that are more reliable are prohibitively costly for employers to acquire. The expected gains from having a reliable source of information do not outweigh the additional costs associated with sampling from that source. Of course, it would be beyond the scope of this paper to explore in detail this alternative explanation for statistical discrimination.

determining the relative costliness of balanced and unbalanced mechanisms. As before, suppose there are two Pareto rankable equilibria, with all workers investing  $y$  and  $z$ , respectively, with  $y < z$ . There exists a critical level of the aggregate,  $f^*$ , that is the policy-maker's goal. The critical strategy for the balanced mechanism,  $x^*$ , is determined by,

$$(5) f^* = \left( \int_0^1 x^{*a} di \right)^{\frac{1}{a}} \Rightarrow x^* = f^*$$

Note that  $x^*$  does not depend upon  $\mathbf{a}$ , due to the (assumed) homogeneity of the employer belief function. The critical mass for any unbalanced mechanism with target worker investment,  $\tilde{x}$ , is determined by,

$$(6) f^* = \left( \int_0^{k^*(\tilde{x})} \tilde{x}^a di + \int_{k^*(\tilde{x})}^1 y^a di \right)^{\frac{1}{a}} \Leftrightarrow k^*(\tilde{x}) = \frac{f^{*a} - y^a}{\tilde{x}^a - y^a}$$

Under this parameterization, therefore,  $k^*(\tilde{x})$  is a continuously differentiable function from  $X$  into  $[0,1]$ .

If  $c(y,x)$  is also smooth we can represent the government's transition minimization problem as in Figure 3. The curve  $g$  represents the function  $c(y, \tilde{x}(k^*))$ , where  $\tilde{x}(k^*)$  is the inverse of  $k^*(\tilde{x})$ . This inverse exists because  $k^*$  is monotonically decreasing in  $\tilde{x}$ . A simple check also verifies that so long as  $c(y,x)$  is not too concave in  $x$ ,  $g$  is convex. The curve  $TC$  represents the total transition cost function,

$$(7) c(y, \tilde{x}) = \frac{TC}{k^*(\tilde{x})}$$

Figure 3 depicts the types of optimal policies that are possible. In (a), the balanced mechanism minimizes transition costs, whilst in (b) an interior solution is depicted. In Figure 3 (c) targeting a small number of workers to use a high strategy is optimal.

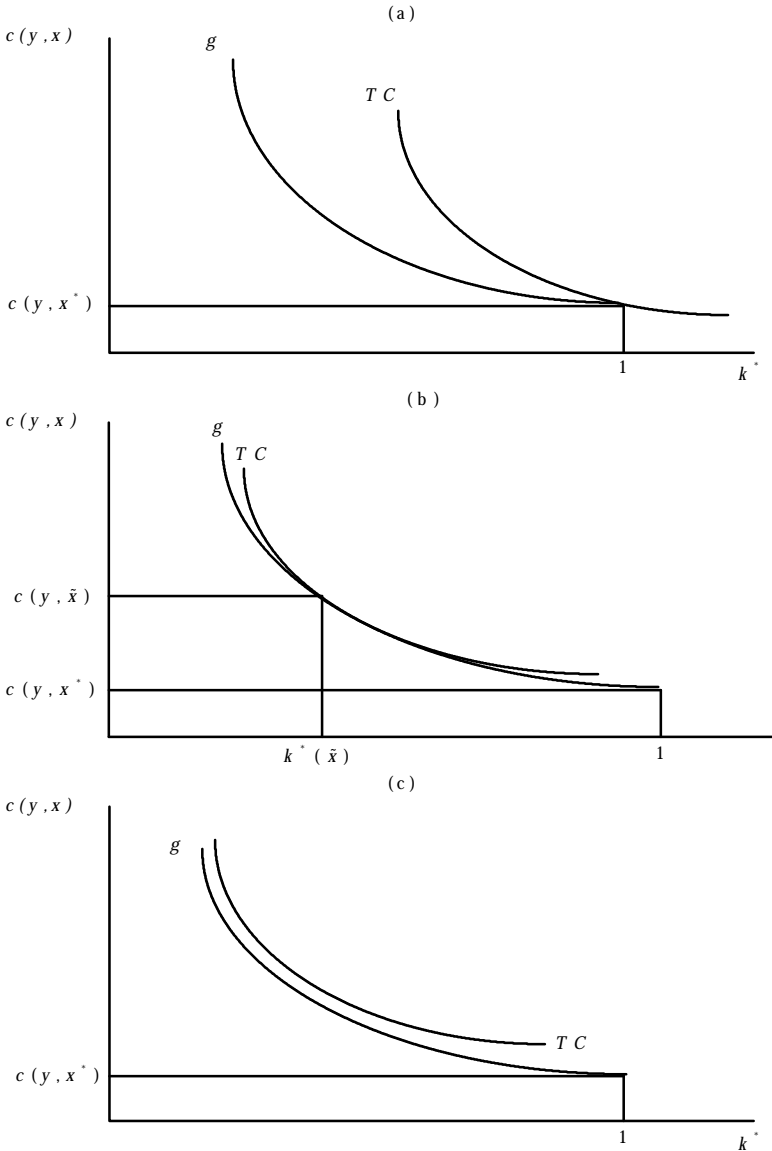


Figure 3. Optimal Policy Choices

What is of interest, however, is how different levels of a change the optimal policy choice of the government. In general,  $c(y, x)$  will not be smooth and may be concave or convex. Nonetheless, even without such

restrictive assumptions, the following comparative statics result is still available.<sup>14</sup>

**Theorem 2.** *In symmetric games with aggregate strategic complementarities and a CES aggregator, the degree of unbalance in the cost minimizing transition policy is nondecreasing in  $a$ .*

**Proof.** *First, observe that  $k^*$  is decreasing in  $a$  and that it is submodular in  $(\bar{x}, a)$ . We need to show that  $\frac{\partial k^*(\bar{x})}{\partial a} c(y, \bar{x})$  is decreasing in  $\bar{x}$ . That this is so follows from the submodularity of  $k^*$  and the assumption that  $c$  is increasing in  $x$ . Thus, total transition costs are submodular in  $(\bar{x}, a)$ . By Topkis' Monotonicity Theorem (Milgrom and Shannon 1994), the optimal choice of  $\bar{x}$  is increasing in  $a$ . The theorem follows from the fact that  $k^*$  is decreasing in  $\bar{x}$*

The above theorem identifies the characteristics of employer beliefs as a significant determinant of the government's cost minimizing choice of transition mechanisms. In situations where beliefs are relatively inflexible, in order to generate the critical aggregate in a cost effective manner, resources have to be spread thinner with the mechanism being more balanced. Thus, the particular transition policy may differ across different labor markets and even different regions. For example, in some markets (e.g., popular music performers), beliefs may be fairly flexible, whilst in others (e.g. CEO positions for women), beliefs could well be extremely rigid. But the overriding conclusion is that, regardless of other potential characteristics of payoff functions, the more inflexible beliefs are, the wider the front one has to push to generate the critical aggregate and break the hold of accommodation.

Parameterizations of the aggregate aside there are other characteristics of economic significance that could influence the nature of the transition mechanism chosen by the government. For example, the actual level of the critical aggregate, while the target of all transition mechanisms, could influence the relative costs of mechanisms. Indeed, it is simple to check that the cost minimizing target strategy is nondecreasing in  $f^*$  and hence, a higher critical aggregate implies a more unbalanced mechanism.

What factors might cause the critical aggregate be greater? The Matsuyama adjustment process discussed in Section III and the analysis of Krugman (1991a), are suggestive of the notion that as agents become more farsighted or as the ability to adjust strategy choices becomes greater, the

<sup>14</sup> The homogeneity of the aggregator is not critical for this result and the theorem would still hold if  $\bar{x} = \int_0^1 x_i^a di$ .

basin of attraction of the high equilibrium shrinks. This means that the critical aggregate that ensures convergence a successful escape is greater and hence, a more unbalanced mechanism will be preferred.

Another potential influence on the choice of mechanism is the level of individual increasing returns. If the total costs associated with raising personal investment rise at a slower rate, then the marginal increase in individual transition costs associated with a higher target strategy could be lower. This could be the case because of initial startup costs in improving human capital. Thus, we could imagine individual transition costs to be some function  $c(y,x;b)$  where  $b$  parameterizes the marginal cost increase of a higher  $x$ . It is straightforward to show that lower levels of  $b$  imply a greater target strategy and hence, a more unbalanced mechanism. Thus, greater individual returns to scale of investment will tend to cause the government to favor a more unbalanced mechanism.

In summary, in this section, it has been shown that the government will find a more unbalanced mechanism to be cost minimizing the more flexible are employer beliefs, the higher the critical aggregate is, and the more personal investment is characterized by individual increasing returns to scale. Indeed, for a resource starved policy-maker, paying attention to these economic characteristics may determine what type of successful escape is feasible at all.

## 5 Conclusions and Extensions

The particular policy choices facing a government in its coordination role have been neglected by economists until now. This paper has highlighted, within the context of a model of statistical discrimination in the labor market, some of the issues confronting governments attempting to change an economy or game from one equilibria to another. In so doing, the possibility of escape (under adaptive expectations) was identified. Moreover, the notion that individual agents need to be convinced to change their behavior by a sufficient amount to break the hold of accommodation was emphasized. In such situations, the policy choices facing a government involve how best to change individual behavior to move them out of the basin of attraction of the low equilibrium. Note though that these considerations apply to many models with public good elements that exhibit strategic complementarities, beyond the application discussed here.

Given these findings, this paper has turned to focus on whether a government should target a small number of individuals for change or adopt a wider coverage. In the context of discrimination in the labor market, the flexibility of employer beliefs and individual returns to scale were singled out as critical determinants of whether a balanced or unbalanced mechanism was cost minimizing. But the assumptions of the game presented abstract

from other factors that might affect the choice of balanced or unbalanced change.

First, the worker's strategy choice is currently assumed to be unidimensional. One could imagine that it is more complex than this. Indeed, it may be multidimensional taking into account differing aspects of education and other investments that improve productivity. Much of the analysis will continue to hold if vector of strategy variables available to an agent were in turn complementary with one another (Milgrom and Roberts 1990). If this is the case, one could then expand the range of mechanisms available for change. For instance, one might have unbalanced mechanisms that attempt to change an individual's behavior on all dimensions or perhaps target some variables by a large amount, leaving the rest to adjust by later momentum.

Second, the game as presented is symmetric in that all workers possess the same payoff functions, face the same aggregate and have the same impact on the aggregate. Introducing heterogeneity complicates matters by making the definitions of balanced and unbalanced mechanisms less clean. Suppose, however, that agents only differed in their costs of investment. That is,  $\pi_i = w(t, \bar{x}) - \gamma_i x$ , where  $\gamma_i$  is the individual investment cost. In that case, since all agents make the same contribution to aggregate employer beliefs, it is clear that the definition of a balanced mechanism remains the same, while the least cost unbalanced mechanism would have those with the lowest investment costs being targeted first. This would continue to be the case if those with the lowest costs also had the greatest impact on the aggregate. However, if this were not the case, then a tradeoff between low costs and greater contributions to the aggregate exists. Nonetheless, a complete analysis of the issues of the optimal composition of a critical mass lie beyond the scope of the dissertation.

Throughout the paper the government was assumed to be a purely external facilitator of change and to possess an external source of resources. Government intervention did not change, permanently, the best response correspondences of agents or if it did, it did so positively. But it is well documented from social psychology that social interventions can have unintended negative feedback effects. In the 1930s, what has become known as the Cambridge-Somerville experiments were conducted in impoverished neighborhoods near Boston. The idea was to take a selection of children from these neighborhoods and devote (virtually) unlimited social work and other resources to helping them escape poverty traps. Thus, education and health care were provided. Regular visits, monitoring and counseling was provided by social workers. Basically, all the ingredients that one could hope for. Nonetheless, years later, the basic social statistics for measuring an

enhanced socioeconomic condition showed little improvement and, on some dimensions, a worsening of children's situations. A suggested reason for the failure of the intervention was an unintended effect: children receiving aid suffered from isolation from their peers. The negative effects of this were enough to cancel the positive benefits of the intervention.<sup>15</sup>

Such considerations, also documented in the case of discrimination by Steele (1992), were not part of the above analysis. In the analysis of this paper, all costs of achieving individual change were subsumed in the function  $c(y,x)$ . Negative feedbacks would enter into these costs and their identification can be of considerable importance for achieving a successful transition. In addition, the discussion in this paper assumes individual transition costs to be independent of the actions of other agents. Fruitful extensions would include relaxing this assumption.

Finally then, the government is currently assumed to know with certainty the critical aggregate, strategies and masses to provide the basis of escape and the real individual transition costs. As is indicated by the previous paragraphs, an interesting and important extension would be to explore more completely relaxations of this assumption and this might change the choice between balanced and unbalanced mechanisms.

By examining the flexibility of employer beliefs, this paper has highlighted the degree of substitutability of individual workers' investments in altering employer beliefs as determining how widespread a government's policy to eliminate discrimination ought to be. This is because, regardless of how they are achieved, once employer beliefs reach some critical level, the discrimination will be removed by a momentum of attrition of stereotypes and of improvements in workers' incentives to take investments that improve their own marginal productivity. Therefore, the costs and tradeoffs associated with changing the behavior of workers to raise employer beliefs to this critical level is the appropriate focus of the government. Such issues are also present in other situations of coordination failure and the choices facing policy-makers in facilitating transition are, in that respect, similar. Nonetheless, this paper represents only a first step in understanding policy in the face of multiple equilibria and in other economic contexts, the critical variables driving choices may well be different and more complex.

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<sup>15</sup> For a discussion of this experiment see Ross and Nisbett (1991)

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