

# Concentration-Based Merger Tests and Vertical Market Structure

*by*

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This paper derives a concentration measure for markets with multiple vertical segments. The measure is derived using a model of vertical contracting where upstream and downstream firms bargain bilaterally and may be integrated. The resulting vertical Hirschman-Herfindahl Index provides a measure of the degree of distortion in the vertical chain as a result of both horizontal concentration in a segment and the degree of vertical integration. Utilisation of this measure would allow competition authorities to distinguish between the differing competitive impacts of upstream and downstream competition, the relative size of integrated firms in each segment and to provide a quantitative threshold test for vertical mergers. *Journal of Economic Literature* Classification Number: L40.

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## 1. Introduction

The use of market share data to form concentration ratios or indexes have become a staple of ‘safe harbour’ tests for horizontal merger analysis. The US Department of Justice bases its measure of concentration on the Hirschman-Herfindahl Index (or *HHI*). This index takes that sum of the squares of the market shares as a measure of concentration. The test has two parts. First, the post-merger *HHI* is calculated and the level of concentration of the market assessed. Markets with an *HHI* above 1800 are said to be highly concentrated, those between 1000 and 1800 moderately concentrated and those below 1000 not concentrated at all. Second, based on the level of concentration, the pre- and post-merger *HHI*'s are compared. For unconcentrated markets, any merger is permissible. However, for moderately and highly concentrated markets, only changes in the *HHI* of less than 100 and 50 respectively will usually be immediately cleared. For mergers outside of these ranges, the DOJ will conduct further analysis as to the merger's competitive effects.

There are many reasons why the *HHI* might be used as a measure of concentration in competition settings.<sup>1</sup> However, the rationale that lies at the heart of the present paper is grounded in the theory of Cournot oligopoly. If we take the Lerner index (that is, price less marginal cost divided by price) as a measure of the level of welfare distortion in a market, in a Cournot equilibrium, the average Lerner index across firms is the *HHI* divided by the price elasticity of market demand. In this sense, the degree to which a

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<sup>1</sup> Perhaps the earliest formal derivation of the *HHI* in a model of oligopoly was by Stigler (1964). He modeled the likelihood that a cartel might be unstable in terms of the probability a deviating member of that cartel might attract a disproportionate share of customers in its favour. That share was inversely related to the *HHI* implying that for greater levels of concentration a cartel might be expected to be more stable. I thank the editor for pointing out this reference.

merger increases the *HHI* is an indication of the degree to which that merger reduces welfare.

This use of the *HHI* has been subject to a number of criticisms. First, if two firms with pre-merger market shares of  $s_1$  and  $s_2$  are analysed, the increase in the *HHI* is usually assessed to be  $2s_1s_2$ . This involves an implicit assumption that the sum of market shares of the merged firms will not change as a result of the merger. However, if this was really the case, the merger would involve no welfare detriment; something that requires the merged parties to exercise market power by contracting their market shares. Only by using a full equilibrium model can one properly assess a merger's impact (Farrell and Shapiro, 1990). Second, and on related lines, this analysis does not consider why a firm's market share may be what it is in the first place. Typically, a large market share implies lower production costs and hence, the reallocation of output following a merger may, in fact, be welfare enhancing (Demsetz, 1974). Finally, competition authorities also have information that flows from the fact that if a merger is proposed, it is likely profitable for the merging parties. This suggests that refinements in the way market shares are used to infer anti-competitive effects can be utilised.<sup>2</sup>

These critiques have not deterred competition authorities from using the *HHI* as a threshold test for the desirability of a merger. In part, this reflects the fact that this use of the *HHI* represents a conservative threshold test in that, if a merger fails to pass it, further analysis of the full equilibrium effects would be possible. In addition, such an equilibrium analysis would be able to take into account other market and technological conditions that may favour one oligopoly model over another.

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<sup>2</sup> The seminal paper in this stream is Farrell and Shapiro (1990), but Daughy (1990), Levin (1990) and McAfee and Williams (1992) offer alternative perspectives. Fels, Gans and King (2000) show how this analysis could inform negotiated undertakings between competition authorities and merger parties.

Nonetheless, with one recent exception, all of these analyses of the competitive effects of mergers and the appropriate use of market shares as threshold indicators of concern neglect the vertical structure of markets. This is somewhat surprising as many competition authorities believe that increasing levels of vertical integration in the market can give rise to anti-competitive concerns.<sup>3</sup> Such beliefs suggest that the level of concentration in a single vertical segment may not reflect the level of anti-competitive potential arising from mergers in and across that segment and that changes in such concentration may not capture the full anti-competitive impacts of a merger. Hence, it would be desirable to have a measure of concentration that reflected vertical issues in markets.

Given this, in this paper, I consider concentration-based tests that take into account the degree to which merged parties are vertically integrated. In so doing, I utilise recent developments in the theory of vertical contracting that gives a general approach to the competition issues that arise from vertical integration. Those developments describe the nature of competition when contracting over input supply terms are negotiated and, in so doing, demonstrate how vertical integration can be utilised as a means of leverage market power across vertical segments. Importantly, as I will show, this theory gives rise to a natural Cournot-type equilibrium outcome that makes it possible to derive appropriate concentration indexes readily comparable with the *HHI* (and indeed collapsing to it in a special case).

The recent theory of vertical contracting was a reaction to the Chicago School critique of vertical merger analyses that stated that integration could not be an instrument

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<sup>3</sup> The US DOJ (1987) and the Australian Competition and Consumer Commission (1994) are explicit in their acknowledgement that mergers that increase the level of vertical integration can be undesirable.

for the leverage of market power as firms with such power could leverage that power through arms-length contracting arrangements and non-linear pricing. Hart and Tirole (1990) were the first to develop a special model that demonstrated that when an upstream monopolist negotiated with downstream firms bilaterally and bilateral agreements could not easily be observed by outside parties then a vertically separated monopolist would be constrained to offer supply terms that dissipated monopoly rents downstream. Put simply, each downstream firm did not trust the monopolist to offer supply terms consistent with a monopoly outcome and the monopolist could not commit to those terms publicly. The end result was an oligopolistic outcome across the industry despite the existence of market power in the upstream segment. This baseline result was subsequently demonstrated to be robust to alternative assumptions on competitive instruments (O'Brien and Shaffer, 1992), information (McAfee and Schwartz, 1994; Segal, 1999; Rey and Verge, 2004), contracting instruments (McAfee and Schwartz, 1994; Segal and Whinston, 2003), contract timing (Gans, 2006), bargaining power (de Fontenay and Gans, 2005) and the presence of upstream competition (de Fontenay and Gans, 2005).

In this environment, vertical integration is a means of restoring industry-wide monopoly outcomes (Rey and Tirole, 2003). Put simply, rent dissipation occurred because an upstream firm was tempted to offer downstream firms secret discounts; imposing negative competitive externalities on other downstream firms. That incentive is mitigated when the upstream firm is integrated downstream as such secret discounts to independent firms harms its own integrated unit. In some cases, the integrated firm has no incentive to supply inputs to other downstream firms and foreclosure and an industry-wide monopoly result. In general, vertical integration, particularly by firms with upstream

or downstream market power, is a means of raising input prices and softening the strength of competition downstream (de Fontenay and Gans, 2005).

In the next section, I take this approach to vertical contracting and integration and use it to derive modified or vertical *HHI* (*VHHI*) that reflects the average degree of Lerner-type distortion across the vertical chain. In so doing, in Section 3, I demonstrate the following: (1) that in the absence of any vertical integration, *VHHI* becomes the *HHI* based solely on downstream market shares; (2) the *VHHI* changes if there are vertically integrated firms who are net input suppliers; (3) the competitive impact of upstream and downstream mergers are distinct; in particular, horizontal mergers amongst non-integrated upstream firms have no impact on the average Lerner index; (4) that vertical mergers that increase downstream market shares or increase the degree to which integrated firms are net suppliers will increase distortions while vertical integration creating a net input demander has no impact on the *VHHI*. Finally, I argue that the *VHHI* offers a more appropriate basis for a threshold test based on market shares than the current *HHI*.

Of course, while the recent theory of vertical contracting and integration yields an elegant, consistent and general theory – and moreover, a simple *VHHI* based on general demand and technology assumptions – there are other theories of vertical relations in the literature. The most prominent of these involves upstream firms setting simple posted prices to downstream firms and firms in each vertical segment compete as Cournot oligopolists. Downstream firms then compete on the basis of these prices or, if they are vertically integrated, on the basis of marginal cost (Salinger, 1988). Vertical integration, therefore, involves potential competitive effects but also efficiency gains as the

successive mark-up or double marginalisation problem is eliminated. For this reason, in Section 4, I consider a vertical concentration measure based on a model of successive Cournot oligopoly. With some assumptions on demand and technologies, this is able to yield a concentration measure that – while more informationally burdensome than the *VHHI* – is related to it and can potentially be applied in regulatory settings. In Section 5, I compute both concentration measures to consider the analysis of the competitive impact of the Exxon-Mobil merger on the Californian petroleum market. A final section concludes.

As alluded to earlier, one other paper considers concentration tests taking into account vertical structure and integration. Hendricks and McAfee (2005) provide an alternative model of outcomes in wholesale markets with many upstream and downstream firms. Based on supply function equilibria models, they focus on the ability of upstream firms to exercise market power and downstream firms to exercise monopsony power and use this to derive an index of equilibrium distortion in the wholesale market. Their analysis identifies the balance between integrated firms' input supply and demand as critical in creating any Lerner-type distortions and derive a modified *HHI* that reflects this.

The main difference between their approach and the standard vertical contracting literature is that their model design offers a means of uncovering a single market clearing linear price when upstream and downstream firms exercise market power (similar to the simple posted prices vertical model). In contrast, the vertical contracting literature focuses on environments where wholesale markets are governed by sets of bilateral negotiations that permit non-linear prices. Not surprisingly, this latter approach leads to

no Lerner-type distortion in the wholesale market taken on its own; the very distortion Hendricks and McAfee set out to quantify. In reality, the difference in approaches corresponds to differences in the type of wholesale market being modeled. Hendricks and McAfee consider general mass markets for inputs where downstream firms may or may not compete directly whereas the vertical contracting literature focuses on inputs supplied to competing downstream firms where input terms are formed by negotiations rather than posted prices.

## 2. Baseline Model and Concentration Index

Here I provide a model of vertical contracting. It is based on de Fontenay and Gans (2004) who consider bilateral bargaining between two upstream firms and two competing downstream firms. Unlike other models, this structure allows for competition in multiple vertical segments and hence, is an appropriate basis for the consideration of a concentration issues across segments. It represents a strict generalisation, in terms of both firm numbers and the nature of upstream and downstream production technologies, over existing models in the literature.

Suppose there are  $N$  firms in an industry indexed  $i = 1, \dots, N$ . Each firm (potentially) operates in an upstream and a downstream vertical segment. Firm  $i$ 's downstream market share is  $s_i$  while its upstream market share is  $\sigma_i$ . The products of firms in the downstream market are perfect substitutes from a final consumer perspective. The (inverse) market demand for that final good is denoted by  $P(Q)$ ; with the usual properties where  $Q$  is total downstream production. For simplicity, I will assume that  $Q$  is simply the sum of all upstream inputs; although all of the results below go through

without this assumption. Firm  $i$ 's upstream costs are a continuously differentiable function,  $C_i(\cdot)$  while its downstream costs (net of input payments) is a continuously differentiable function,  $c_i(\cdot)$ . Notice that, in principle, while final goods are homogeneous, intermediate inputs may not be and integrated firms may have a lower or higher cost structure than non-integrated ones.

### *Bilateral Bargaining*

I follow the standard timeline in the recent vertical contracting literature:

STAGE 1 (*Bargaining*): Bargaining over input supply terms takes place between each firm.

STAGE 2 (*Production*): Production takes place and payoffs are realised.

As in the vertical contracting literature, it is assumed that there are a set of bilateral Nash bargaining games between upstream and downstream pairs. Each upstream-downstream pair negotiates over price and quantity supply terms. For example,  $i$  and  $j$  bargain over terms specifying a quantity of inputs purchased,  $q_{ij}$ , and a lump-sum transfer,  $p_{ij}$  paid by  $i$  to  $j$ .

The precise game theoretic relationship between the set of Nash bargains is not modeled here. Those negotiations could be simultaneous (as in Segal, 1999; and O'Brien and Shaffer, 2004) or sequential with passive beliefs (as in McAfee and Schwartz, 1994; and de Fontenay and Gans, 2005). Either approach leads to the same outcome with regard to input quantities traded: that pairwise negotiations between firms over input supply terms will satisfy *bilateral efficiency*. That is, when pairs cannot contract or observe the outcomes of other negotiations during their own, there exists an equilibrium where they undertake those negotiations holding the outcomes of others as fixed. This means that the

quantity of inputs traded will be such that the *joint profits of both parties are maximised* holding fixed the quantity of inputs expected to be traded as a result of other pairwise negotiations. It is this equilibrium, which is the main focus of the vertical contracting literature, which will be the focus of this paper.

### *Lerner Index for a Vertical Chain*

Consider a representative negotiation between firms  $i$  and  $j$  over  $q_{ij}$  (that quantity supplied by  $j$  to  $i$ ) and  $p_{ij}$  (the payment from  $i$  to  $j$ ). That is, in this negotiation,  $i$  is the downstream firm while  $j$  is the upstream firm (even though each potentially has operations in the other vertical segment). The profits of  $i$  and  $j$  (written to highlight this quantity and price) are:

$$\pi_i = \underbrace{P(Q)\left(q_{ij} + \sum_{k \neq j} q_{ik}\right) - p_{ij} - \sum_{k \neq j} p_{ik} - c_i(q_{ij}, \cdot)}_{i\text{'s Downstream Profits}} + \underbrace{\sum_k p_{ki} - C_i(\cdot)}_{i\text{'s Upstream Profits}} \quad (1)$$

$$\pi_j = \underbrace{P(Q)\sum_k q_{jk} - \sum_k p_{jk} - c_j(\cdot)}_{j\text{'s Downstream Profits}} + \underbrace{p_{ij} + \sum_{k \neq i} p_{kj} - C_j(q_{ij}, \cdot)}_{j\text{'s Upstream Profits}} \quad (2)$$

Bilateral efficiency implies that the two firms will agree to a quantity ( $q_{ij}$ ) that maximises the sum of (1) and (2) taking as given all other input prices and quantities. This is equivalent to solving:

$$\max_{q_{ij}} P(Q)\left(q_{ij} + \sum_{k \neq j} q_{ik}\right) - c_i(q_{ij}, \cdot) + P(Q)\sum_k q_{jk} - C_j(q_{ij}, \cdot) \quad (3)$$

This implies that:

$$\begin{aligned} P(Q) + P'(Q)\sum_k (q_{ik} + q_{jk}) - \frac{\partial c_i}{\partial q_{ij}} - \frac{\partial C_j}{\partial q_{ij}} &= 0 \\ \Rightarrow \frac{P - \frac{\partial c_i}{\partial q_{ij}} - \frac{\partial C_j}{\partial q_{ij}}}{P} &= -\frac{P'(Q)\sum_k (q_{ik} + q_{jk})}{P} = \frac{1}{\varepsilon}(s_i + s_j) \end{aligned} \quad (4)$$

where  $\varepsilon = -P/P'Q$  is the market price elasticity of demand. This is the Lerner index for

the entire vertical chain in this model. Notice that it depends only on downstream market shares and not upstream shares. The intuition for this is that, in bilateral negotiations, both firms take into account impacts of changes in input supply between them on their downstream profits and this depends on their downstream market shares. The only upstream impacts come through marginal costs but these depend upon the absolute (rather than relative) level and nature of their upstream outputs.

### *Vertical Hirschman-Herfindahl Index*

Let  $HHI \equiv \sum_{i=1}^N s_i^2$ . Utilising (4), the average Lerner index for the industry can be derived.

**Proposition 1.** *The average Lerner index is  $\frac{1}{\varepsilon} \sum_{i=1}^N s_i (s_i + \sigma_i - q_{ii} / Q) = \frac{1}{\varepsilon} HHI + \frac{1}{\varepsilon} \sum_{i=1}^N s_i (\sigma_i - q_{ii} / Q)$ .*

All proofs are in the appendix. Here  $q_{ii}$  is the level of internal supply. Notice that the higher is this, the lower is the average Lerner index. This is because internal supply simply maximises individual profits as opposed to bilateral profits that are maximised in determining other supply terms and so there is no internalisation of competitive externalities in this case. Thus, as  $q_{ii}$  ranges from 0 to  $\min\{\sigma_i, s_i\}Q$ , for all  $i$ , the average Lerner index moves from  $\frac{1}{\varepsilon} HHI$  to  $\frac{1}{\varepsilon} \sum_{i=1}^N s_i \max\{s_i, \sigma_i\}$ .

Indeed, there is a sense in which this latter measure provides a solid basis for an appropriate concentration measure.

**Corollary 1.** *Suppose that, for all  $i$ ,  $q_{ii} = \min\{s_i, \sigma_i\}Q$ . The average Lerner index is  $\frac{1}{\varepsilon} \sum_{i=1}^N s_i \max\{s_i, \sigma_i\}$ .*

The assumption here implies that if  $s_i \geq \sigma_i$  then  $\sum_{j \neq i} q_{ji} = 0$  and if  $s_i \leq \sigma_i$  then  $\sum_{j \neq i} q_{ij} = 0$ . It amounts to an assumption that each firm does not care about the source of its input supply but that if it provides inputs into this market, then it will demand those inputs first before sourcing them from others.

From this corollary, it is easy to see that the appropriate concentration index for this type of model is,  $VHHI \equiv \sum_{i=1}^N s_i \max\{s_i, \sigma_i\}$ . As an index it shares with the simple structure of  $HHI \equiv \sum_{i=1}^N s_i^2$  being a sum of squared market shares and a value between 0 (in the case of perfect competition in both vertical segments) and 10,000 (in the case of a downstream monopolist). As such, it is readily comparable to the thresholds established for the application of the  $HHI$ ; including equal firm size equivalency comparisons.

It is perhaps instructive, at this point, to consider the differences between the  $HHI$  and  $VHHI$  by way of a simple example. Imagine that there is one vertically integrated firm, two independent downstream firms and one independent upstream firm. Suppose that all firms are symmetric within their segment and that downstream firms have no costs (other than input payments) while upstream firms have cost functions of the form,  $C(\cdot) = q_j^2$ . Let final market demand is linear,  $P(Q) = 1 - Q$ . In this situation, prior to any merger, it is straightforward to calculate that the integrated firm will not supply independent downstream firms in equilibrium and will produce output of  $\frac{5}{26}$  while the independent upstream firm will supply  $\frac{6}{26}$ ; divided equally amongst the two downstream firms. Thus,  $P = \frac{15}{26}$ . If, however, independent upstream firm merged with one of the independent downstream firms, it is easily to calculate that, following this merger, the remaining independent downstream firm would not receive any supply in equilibrium.

The two integrated firms would split the market and each supply  $\frac{1}{5}$  leading to a price of  $\frac{3}{5}$ ; a lower quantity and higher price than prior to the merger.

The interesting thing about this example is that traditional merger analysis (where the pre-merger shares of the merging firms are summed and concentration measures calculated), utilising the *HHIs* only would not have revealed any issue. The upstream and downstream *HHIs* are 5041 and 3353 respectively both pre- and post-merger. In contrast, the *VHHI* rises from 3353 (the same as the downstream *HHI*) prior to the merger to 4298. Thus, utilising it would have identified concerns worthy of closer examination.

### **3. Implications for Merger Analysis**

In this section, I consider the implications of utilising *VHHI* for the purpose of merger analysis. In so doing, downstream mergers, upstream mergers and vertical mergers are evaluated in turn.

#### *Downstream Mergers*

When all downstream firms in an industry are net buyers of upstream inputs,  $VHHI = HHI$  and horizontal mergers will appropriately be evaluated using the *HHI*. Vertical separation of all downstream firms or, conversely, the lack of external trade between integrated firms would similarly satisfy this condition.

If some firms are integrated and net suppliers of upstream inputs, then for the purposes of measuring post firm concentration,  $VHHI > HHI$ . Put simply, in this situation, downstream competition is unlikely to follow a pure Cournot outcome and so the *HHI* understates the level of concentration. Nonetheless, even in this situation, if two

firms,  $i$  and  $j$ , merge who are net buyers, then the change in  $VHHI$  will be  $2s_i s_j$ ; the same as it would be using the  $HHI$ .

In these cases, merger analysis will primarily focus on downstream market shares. It is only where at least one of the merging firms is a net supplier of upstream inputs that the change in concentration will be  $s_i(s_j + \sigma_j)$  (if only  $j$  is a net supplier) and  $s_i\sigma_j + s_j\sigma_i$  (if both  $i$  and  $j$  are net suppliers). In both of these cases, the upstream shares of one or both firms become relevant in evaluating the merger. In this situation, it is simply not possible to use a functional market separation along vertical lines to evaluate the impact of the merger.

### *Upstream Mergers*

When input supply terms are determined by bilateral bargaining, a clear implication is that, under vertical separation, upstream market structure does not matter for overall quantity and price downstream. This is a direct implication of (4) and a generalisation to the case of upstream competition of results that non-integrated upstream monopolies are unable to leverage their market power downstream.<sup>4</sup> Recall, that when firms are not integrated (or more generally integrated firms are net buyers of inputs) the average Lerner index for the whole vertical chain does not depend on upstream market shares. Hence, should upstream firms merge, the  $VHHI$  would be unchanged.

While at a broad level this suggests that competition authorities should view purely upstream and purely downstream mergers differently, when there is vertical

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<sup>4</sup> This outcome is contained in both de Fontenay and Gans (2004) and O'Brien and Shaffer (2004). The latter paper then considers how restrictions on the ability of multi-product firms to bundle may give rise to welfare effects from upstream mergers.

integration, the strong result that upstream mergers (absent other efficiencies) are welfare neutral is potentially weakened. For example, if a vertically integrated firm  $i$  and a non-integrated upstream firm  $j$  merge and  $\sigma_i + \sigma_j > s_i$ , then the change in the  $VHHI$  as a result of that merger will be  $s_i(\max\{\sigma_i - s_i, 0\} + \sigma_j)$ . Thus, the greater the upstream market share of the merging firms, the greater the increase in  $VHHI$ . In addition, the greater the degree of vertical integration amongst merging firms, the greater the potential competition concern from upstream mergers.

### *Vertical Mergers*

While the above analysis indicates the potential changes in horizontal merger analysis based on alternative vertical market structures it is in the analysis of vertical mergers that the  $VHHI$  is at its potentially most useful. To date, competition authorities have not been able to provide bright-line safe harbour tests for vertical mergers. The USDOJ (1984) alludes to the degree of concentration in a vertical segment as being of issue in its evaluation but there is no further guidance beyond this. To be sure, the level of upstream competition does, in fact, play an important role in mitigating adverse competitive consequences from vertical mergers (de Fontenay and Gans, 2004). But precisely how much competition is required for this has to date been unknown.

The  $VHHI$  provides guidance on this front. First, it provides a baseline measure of the level of relevant concentration over the entire vertical chain to determine whether an industry facing a vertical merger should be considered concentrated or not. Second, it suggests that the nature of the vertically integrated firm; that is, whether it ends up a net supplier of inputs or not is important. Finally, it provides a way of considering mergers

between firms with differing degrees of vertical integration.

To see this, let's begin with a situation where no firm in an industry is integrated. A merger between any upstream and downstream firm is *pure* vertical integration. Imagine that there are 4 equal sized upstream firms and 10 equal sized downstream firms. In this case, prior to the merger the upstream *HHI* is 2500, the downstream *HHI* is 1000 as is the *VHHI*. If one upstream and one downstream firm merge, the upstream and downstream *HHI*'s remain unchanged whilst the post-merger *VHHI* becomes 1150. In this case, if we applied the same thresholds as the USDOJ utilising the *VHHI*, this would be regarded as a moderately concentrated industry and hence, the merger would violate those thresholds.

In contrast, imagine that there are 8 downstream firms with market shares of 10% each and an additional firm with a market share of 20%. In this case, if that larger downstream firm should merge with an upstream firm, the pre- and post-merger upstream and downstream *HHI*'s would be 2500 and 1300 respectively while the pre- and post-merger *VHHI*'s would be 1300 and 1400. Utilising the USDOJ thresholds, this merger – again in a moderately concentrated industry – would just satisfy the threshold for a safe harbour.

Thus, despite effectively a higher presumptive level of concentration in each vertical segment *and* a merger creating a significantly larger firm in the second example that merger is potentially less anti-competitive. The reason is that while at least 60 percent of the integrated firm's output will be sold to other downstream firms in the first case, only 20 percent will be sold to those firms in the second. Thus, the potential for negotiations with those firms to have a significant overall effect on competition is much

lower.

These examples demonstrate the usefulness of the *VHHI* measure. Not only does it take into account upstream and downstream competition where relevant but it also takes into account the likely position of the vertically integrated firm. When that firm is not a significant net supplier of inputs, then the likely anti-competitive effects arising from it are likely to be lower.

But the *VHHI* also allows us to consider more carefully the overall industry-wide effects of a vertically integrated and a non-integrated firm. For instance, building on the first example, suppose that the vertically integrated firm there (with a 10% downstream and 25% upstream share) was to merge with another downstream firm. In this case, because the only segment both firms operated in would be the downstream segment, it is likely that competition authorities would evaluate the merger on that basis. In that case, the downstream *HHI* would change from 1000 to 1300 and be regarded as presumptively anti-competitive.

In contrast, using the *VHHI*, the merger would change it from 1150 to 1400. While still presumptively anti-competitive, the magnitude of the change in concentration is significantly less. Put simply, the nominally horizontal merger makes the integrated firm a relatively smaller net supplier and this effect mitigates the usual anti-competitive concerns based on an analysis of concentration in a single segment.

#### **4. Concentration Measures in Successive Cournot Oligopoly**

In some markets, firms may not be able to negotiate over non-linear prices and may be constrained to offer linear ones. As is well known, this gives rise to the problem

of double marginalisation: a problem that can be resolved by vertical integration. Consequently, a vertical merger may have an anti-competitive effect of the type described earlier along-side a pro-competitive one in terms of eliminating double mark-ups. It is, therefore, instructive to consider a vertical concentration measure that takes each of these effects into account.

Here I derive a measure of concentration based on the successive Cournot oligopoly model of Salinger (1988). Like Salinger, I impose assumptions of linear demand and costs. The basic timeline of the model is as follows:

**STAGE 1 (*Wholesale Market Competition*):** Upstream firms compete in Cournot quantity competition for the sale of inputs to downstream firms.

**STAGE 2 (*Downstream Market Competition*):** On the basis of wholesale market prices, downstream firms operate as Cournot competitors in competition for final consumers.

Thus, in contrast to the previous model based on bilateral bargaining, input prices are simple per unit prices only and hence, will involve upstream firms earning a marginal above their marginal cost for external sales.

Given this set-up, the following proposition states the analogue to the concentration measure derived in Proposition 1.

**Proposition 2.** *Under linear downstream demand and costs, the vertical HHI in the successive Cournot oligopoly model is:*

$$\frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j^2 + \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 - \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} s_j + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2.$$

Notice that this is the sum of the upstream and downstream *HHI*'s less a term that reflects the lack of distortion for internal trade within a firm and plus a distortion reflecting the concentration of external trade between firms.

There are some interesting things to note about this index. First, if all firms are vertically integrated and identical then it simply becomes the *HHI* for a single segment. Second, if no firm is vertically integrated, the index becomes:

$$\frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j^2 + \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i=1}^N q_{ij}^2 \quad (5)$$

Notice that for the case where there are two upstream and downstream monopolists, this becomes  $3/\varepsilon$ . This implies that the equilibrium level of  $\varepsilon$  would have to be greater than 3. This reflects the multiple distortions arising from double marginalisation. Finally, notice that for a bottleneck monopolist in a segment with perfect competition in the other segment, the index reduces to 1.

Finally,  $x_j = \sum_i q_{ji}$  and consider the following simplification:

**Corollary 2.** *Suppose that, for all  $j$ ,  $q_{jj} = \min[s_j, \sigma_j]Q$  and for all  $(i,j)$ ,  $q_{ij} = \frac{s_i - \sigma_i}{\sum_i \max[0, s_i - \sigma_i]} (q_j - x_j)$ , then*

$$VHHI = \frac{1}{\varepsilon} \sum_{j=1}^N s_j \max[s_j, \sigma_j] + \frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j (\sigma_j - s_j) + \sum_{j=1}^N \sum_{i=1}^N \left( \frac{(s_i - \min[s_i, \sigma_i])(\sigma_j - \min[s_j, \sigma_j])}{1 - \sum_i \min[s_i, \sigma_i]} \right)^2.$$

Thus, this concentration measure is similar to the one based on the contracting model but with the additional distortions from double marginalisation and the additional distortion (or removal of distortion) based on whether a firm is a net supplier (or net buyer) in the wholesale market.

In this model, all other things being equal, horizontal mergers are distortionary while vertical mergers improve efficiency. However, what this allows is for a consideration of these offsetting effects when vertical mergers occur that increase horizontal concentration in either or both upstream and downstream markets. We demonstrate how this applies in the next section.

## 5. Application

As an illustration of how these concentration measures may be useful in providing guidelines for merger analysis, I consider here (as did Hendricks and McAfee, 2005) the impact of the Exxon and Mobil merger on the California petrol retailing market. Hendricks and McAfee (2005) study this market because of its relative isolation to the rest of the United States (for transportation and regulatory reasons). However, their focus is not on concentration measures as a threshold test for competitive concern but on a full analysis of the impact of the merger on intermediate and final good prices. Nonetheless, using the threshold tests here yields similar conclusions.

Table 1 presents information on the market shares of petrol refining and retailing participants in California.<sup>5</sup> Notice that both Mobil and Exxon have larger downstream market shares than upstream ones. Thus, each is a net purchaser in the wholesale petrol market and will remain so following the summation of their market shares.

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<sup>5</sup> The data is from Hendricks and McAfee (2005) who themselves utilise data from unpublished work by Leffler and Pulliam (1999).

**Table 1: Market Shares (Based on Sales)**

<b>Company</b>	<b>Upstream (Refining) Market Share (%)</b>	<b>Downstream (Retailing) Market Share (%)</b>
Chevron	26.4	19.2
Tosco	21.5	17.8
Equilon	16.6	16
Arco	13.8	20.4
<b>Mobil</b>	<b>7</b>	<b>9.7</b>
<b>Exxon</b>	<b>7</b>	<b>8.9</b>
Ultramar	5.4	6.8
Paramount	2.3	0
Kern	0	0.3
Koch	0	0.2
Vitol	0	0.2
Tasoro	0	0.2
PetroDiamond	0	0.1
Time	0	0.1
Glencoe	0	0.1

Table 2 reports various concentration measures. The first two columns are the pre- and post-merger concentration measures based on a simple summation of market shares (as would occur in threshold tests). Notice that, for the first three measures, the threshold requirements for the USDOJ would not be met as either the post-merger measure was highly concentrated or the merger raised the concentration measure by more than 100 points. Notice, however, that the percentage increase in the *VHHI* (Contracting) measure (9.6%) is greater than the *VHHI* (Cournot) measure (7.1%) because the latter involves an efficiency benefit as a greater proportion of wholesale market trade is internal to an integrated firm while the former involves a large increase in downstream concentration; something that causes greater competitive distortions in the contracting model.

**Table 2: Concentration Measures**

<b>Concentration Measure</b>	<b>Pre-Merger</b>	<b>Post-Merger</b>	<b>Post-Merger with Exxon Refinery Divestiture</b>	<b>Post-Merger with Exxon Retail Divestiture</b>
Upstream <i>HHI</i>	1758	1856	1758	1856
Downstream <i>HHI</i>	1572	1739	1739	1572
<i>VHHI</i> Contracting	1786	1953	1953	1833
<i>VHHI</i> Cournot	1963	2103	2157	2097

Divestitures may have also been considered as options to resolve vertical problems. Notice that if Exxon's refinery assets were divested to an independent competitor, then this would resolve an upstream concentration issues (meeting USDOJ thresholds) but both *VHHI* measures exhibit a continuing problem as downstream concentration has increased. In contrast, divestiture of Exxon's retail assets would result in a significantly reduced increase in *VHHI* (Contracting) while being neutral in comparison with a full merger for *VHHI* (Cournot). This suggests that a retail divestiture would be more desirable than an upstream divestiture in this instance. Put simply, the most competitive damage appears here to be coming from the increase in downstream concentration relative to upstream concentration.<sup>6</sup>

This is not to suggest that concentration measures alone should dictate whether a merger should be opposed by competition authorities.<sup>7</sup> Here, however, in establishing threshold guidelines for competitive concern, measures that take into account vertical issues can be very useful in cases where proposed mergers involve parties with market power in one or both vertical segments. Moreover, for mergers that are purely vertical,

<sup>6</sup> Interestingly, concentrating on the wholesale market effects, Hendricks and McAfee's (2005) model suggested that a downstream divestiture would achieve little as the 'balance of trade' between firms would largely be unaltered relative to a full merger. In contrast, an upstream divestiture would bring about a relatively more balanced wholesale market and fewer distortions.

<sup>7</sup> See Hendricks and McAfee (2005) for an argument in favour of broader simulations.

these measures provide a new approach to setting quantitative guidelines.

## **6. Conclusion**

In conclusion, the analysis here demonstrates that the evaluation of mergers involving or creating integrated firms is more nuanced than purely horizontal mergers without any cross-segment impacts. Vertical mergers create anti-competitive concerns through a different path than the unilateral effects created by pure horizontal mergers. In addition, horizontal mergers involving integrated firms can sometimes create outcomes that balance the usual anti-competitive concerns regarding such mergers. The use of the *VHHI* rather than a segment-level *HHI* as the basis for threshold tests captures these differing effects.

Of course, it would also be instructive to build the analysis here into the equilibrium analyses like Farrell and Shapiro (1990). After all, like horizontal mergers in Cournot oligopolies, vertical integration when there is upstream competition may also not be privately profitable (de Fontenay and Gans, 2005). As such, the fact that a vertical merger is proposed contains additional information regarding its likely anti-competitive effects. That type of analysis is, however, left for future work.

## Appendix

### *Proof of Proposition 1 and Corollary 1*

To find the average Lerner index, we take (4) (and where  $i = j$ , the standard Lerner condition  $\frac{1}{\varepsilon} s_i$ ) and multiply it by  $q_{ij}$  and sum:

$$\begin{aligned}
& \frac{1}{Q} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij} \frac{1}{\varepsilon} (s_i + s_j) + \frac{1}{Q} \sum_{i=1}^N q_{ii} \frac{1}{\varepsilon} s_i \\
&= \frac{1}{Q} \sum_{i=1}^N \sum_{j \neq i}^N q_{ij} \frac{1}{\varepsilon} s_i + \frac{1}{\varepsilon} \sum_{j=1}^N (\sigma_j - q_{jj} / Q) s_j + \frac{1}{Q} \frac{1}{\varepsilon} \sum_{i=1}^N q_{ii} s_i \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N (s_i - q_{ii} / Q) s_i + \frac{1}{\varepsilon} \sum_{i=1}^N (q_{ii} / Q) s_i + \frac{1}{\varepsilon} \sum_{j=1}^N (\sigma_j - q_{jj} / Q) s_j \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 + \frac{1}{\varepsilon} \sum_{j=1}^N (\sigma_j - \min\{\sigma_j, s_j\}) s_j \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 + \frac{1}{\varepsilon} \sum_{i=1}^N (\sigma_i - \min\{\sigma_i, s_i\}) s_i \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 + \frac{1}{\varepsilon} \sum_{i=1}^N \max\{0, \sigma_i - s_i\} s_i \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N s_i (s_i + \max\{0, \sigma_i - s_i\}) \\
&= \frac{1}{\varepsilon} \sum_{i=1}^N s_i \max\{s_i, \sigma_i\} \tag{6}
\end{aligned}$$

### *Proof of Proposition 2*

Working backwards, in this model, if  $i$  is non-integrated, its downstream profits are given by:  $P(Q)x_i - c_i(x_i) - p_i x_i$ . Maximising this with respect to  $x_i$  (holding other downstream quantities as given) gives the (inverse) input demand function:  $p_i = P(Q) + P'(Q)x_i - \frac{\partial c_i}{\partial x_i}$  where  $x_i = \sum_j q_{ij}$  (i.e., the sum of inputs purchased from upstream firms, as indexed by  $j$ ).

Upstream firms compete in Cournot for downstream customers based on downstream individual demand. Note that those demands are interdependent. Hence,

$$\frac{\partial p_i}{\partial q_{ij}} = 2P'(Q) + P''(Q)x_i - \frac{\partial^2 c_i}{\partial x_i^2}$$

$$\frac{\partial p_i}{\partial q_{kj}} = P'(Q) + P''(Q)x_i$$

Upstream firms solve:  $\max_{\{q_{ij}\}_i} \sum_{i \neq j} p_i q_{ij} + P(Q)x_j - c_j(x_j) - p_j \sum_{i \neq j} x_{ij} - C_j(q_j)$ . Here we use  $x_j$  to denote  $j$ 's downstream quantity while  $q_j$  denotes its upstream quantity. This gives first order conditions of:

$$\frac{\partial p_i}{\partial q_{ij}} q_{ij} + p_i + \sum_{-i,j} \frac{\partial p_k}{\partial q_{ij}} q_{kj} + P'(Q)x_j - \frac{\partial p_j}{\partial q_{ij}} \sum_{i \neq j} q_{ij} - \frac{\partial C_j}{\partial q_{ij}} = 0 \text{ for all } i \neq j \quad (7)$$

$$\sum_{-j} \frac{\partial p_k}{\partial q_{ij}} q_{kj} + P(Q) + P'(Q)x_j - \frac{\partial c_j}{\partial q_{ij}} - \frac{\partial p_j}{\partial q_{ij}} (x_j - q_{ij}) - \frac{\partial C_j}{\partial q_{ij}} = 0 \text{ for } i = j \quad (8)$$

Note that (7) implies that:

$$\begin{aligned} & \left( 2P'(Q) + P''(Q)x_i - \frac{\partial^2 c_i}{\partial x_i^2} \right) q_{ij} + P(Q) + P'(Q)x_i - \frac{\partial c_i}{\partial x_i} + P'(Q)x_j \\ & + \sum_{-i,j} (P'(Q) + P''(Q)x_k) q_{kj} - (P'(Q) + P''(Q)x_j)(x_j - q_{ij}) - \frac{\partial C_j}{\partial q_j} = 0 \\ \Rightarrow & \sum_{i \neq j} (P'(Q) + P''(Q)x_i) q_{ij} - (P'(Q) + P''(Q)x_j)(x_j - q_{ij}) + P'(Q)x_j \\ & + \left( P'(Q) - \frac{\partial^2 c_i}{\partial x_i^2} \right) q_{ij} + P(Q) + P'(Q)x_i - \frac{\partial c_i}{\partial x_i} - \frac{\partial C_j}{\partial q_j} = 0 \\ \Rightarrow & P(Q) + P'(Q)(q_j - q_{ij} + q_{ij} + x_i) + P''(Q) \left( \sum_{i \neq j} x_i q_{ij} - x_j(x_j - q_{ij}) \right) - \frac{\partial^2 c_i}{\partial x_i^2} q_{ij} = \frac{\partial c_i}{\partial x_i} + \frac{\partial C_j}{\partial q_j} \\ \Rightarrow & P(Q) + P'(Q)(q_j + x_i + q_{ij}) + P''(Q) \left( \sum_i x_i q_{ij} - x_j^2 \right) - \frac{\partial^2 c_i}{\partial x_i^2} q_{ij} = \frac{\partial c_i}{\partial x_i} + \frac{\partial C_j}{\partial q_j} \end{aligned} \quad (9)$$

Similarly (8) implies:

$$\begin{aligned} & \sum_{-j} (P'(Q) + P''(Q)x_k) q_{kj} + P(Q) + P'(Q)x_j - \frac{\partial c_j}{\partial x_j} - (P'(Q) + P''(Q)x_j - \frac{\partial^2 c_j}{\partial x_j^2})(x_j - q_{ij}) - \frac{\partial C_j}{\partial q_j} = 0 \\ \Rightarrow & P(Q) + P'(Q)(x_j + q_j - q_{ij} - x_j + q_{ij}) + P''(Q) \sum_{-j} x_k q_{kj} - (P''(Q)x_j - \frac{\partial^2 c_j}{\partial x_j^2})(x_j - q_{ij}) = \frac{\partial c_j}{\partial x_j} + \frac{\partial C_j}{\partial q_j} \quad (10) \\ \Rightarrow & P(Q) + P'(Q)q_j + P''(Q) \left( \sum_i x_i q_{ij} - x_j^2 \right) + \frac{\partial^2 c_j}{\partial x_j^2} (x_j - q_{ij}) = \frac{\partial c_j}{\partial x_j} + \frac{\partial C_j}{\partial q_j} \end{aligned}$$

It is useful to note that, if downstream demand and costs are linear (i.e.,  $P''(Q) = 0$  and  $\frac{\partial^2 c_j}{\partial x_j^2} = 0$ ), all FOCs are independent of  $x_j$  (that is,  $q_{ij}$  and  $q_{jj}$  do not depend upon  $j$ 's downstream market share).

Let  $\sigma_j$  and  $s_j$  denote  $j$ 's upstream and downstream market shares. From (9) and (10) we can derive the distortion from each quantity decision:

$$\frac{P - \frac{\partial c_i}{\partial x_i} - \frac{\partial C_j}{\partial q_j}}{P} = \frac{1}{\varepsilon}(\sigma_j + s_i + q_{ij}/Q) - \frac{P''(Q)(\sum_i x_i q_{ij} - x_j^2) - \frac{\partial^2 c_i}{\partial q_i^2} q_{ij}}{P} \quad (11)$$

$$\frac{P - \frac{\partial c_i}{\partial x_j} - \frac{\partial C_j}{\partial q_j}}{P} = \frac{1}{\varepsilon}\sigma_j - \frac{P''(Q)(\sum_i x_i q_{ij} - x_j^2) + \frac{\partial^2 c_j}{\partial x_j^2}(x_j - q_{jj})}{P} \quad (12)$$

Note that if downstream demand has a constant elasticity,  $\varepsilon$ , then these become:

$$\begin{aligned} \frac{P - \frac{\partial c_i}{\partial x_i} - \frac{\partial C_j}{\partial q_j}}{P} &= \frac{1}{\varepsilon}(\sigma_j + s_i + q_{ij}/Q) - \varepsilon(1 + \varepsilon)\left(\sum_i (q_{ij}/Q)(s_i - s_j)\right) - \frac{\frac{\partial^2 c_i}{\partial q_i^2} q_{ij}}{P} \\ \frac{P - \frac{\partial c_i}{\partial x_j} - \frac{\partial C_j}{\partial q_j}}{P} &= \frac{1}{\varepsilon}\sigma_j - \varepsilon(1 + \varepsilon)\left(\sum_i (q_{ij}/Q)(s_i - s_j)\right) - \frac{\frac{\partial^2 c_j}{\partial x_j^2}(x_j - q_{jj})}{P} \end{aligned}$$

The vertical *HHI* is constructed by taking a weighted average of the above distortions with respect to each share of the input trade as a function of total output (that is, assigning weights of  $q_{ij}/Q$ ).

Finally, take the weighted sum of the Lerner indexes:

$$\begin{aligned} &\frac{1}{Q} \sum_j \sum_i q_{ij} \frac{P - \frac{\partial c_i}{\partial x_i} - \frac{\partial C_j}{\partial q_j}}{P} \\ &= \frac{1}{Q} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij} \frac{1}{\varepsilon}(\sigma_j + s_i + q_{ij}/Q) + \frac{1}{Q} \sum_{j=1}^N q_{jj} \frac{1}{\varepsilon}\sigma_j \\ &= \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij} \sigma_j + \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i=1}^N q_{ij} s_i - \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} s_j + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2 + \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} \sigma_j \\ &= \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_j \sigma_j + \frac{1}{Q} \frac{1}{\varepsilon} \sum_{i=1}^N \sum_{j=1}^N q_{ij} s_i - \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} s_j + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2 \\ &= \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_j \sigma_j + \frac{1}{Q} \frac{1}{\varepsilon} \sum_{i=1}^N x_i s_i - \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} s_j + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2 \\ &= \frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j^2 + \frac{1}{\varepsilon} \sum_{i=1}^N s_i^2 - \frac{1}{Q} \frac{1}{\varepsilon} \sum_{j=1}^N q_{jj} s_j + \frac{1}{Q^2} \frac{1}{\varepsilon} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2 \end{aligned}$$

*Proof of Corollary 2*

Given these assumptions,

$$\frac{1}{Q^2} \sum_{j=1}^N \sum_{i \neq j}^N q_{ij}^2 - \frac{1}{Q^2} \sum_{j=1}^N q_{ij} x_j = \sum_{j=1}^N \sum_{i=1}^N \left( \frac{(s_i - \min[s_i, \sigma_i])(\sigma_j - \min[s_j, \sigma_j])}{1 - \sum_i \min[s_i, \sigma_i]} \right)^2 - \sum_{j=1}^N s_j \min[s_j, \sigma_j]$$

With this we have

$$\begin{aligned} & \frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j^2 + \frac{1}{\varepsilon} \sum_{j=1}^N s_j^2 - \sum_{j=1}^N s_j \min[s_j, \sigma_j] + \sum_{j=1}^N \sum_{i=1}^N \left( \frac{(s_i - \min[s_i, \sigma_i])(\sigma_j - \min[s_j, \sigma_j])}{1 - \sum_i \min[s_i, \sigma_i]} \right)^2 \\ &= \frac{1}{\varepsilon} \left( HHI_{Up} + HHI_{Down} + \sum_{j=1}^N \left( -s_j \min[s_j, \sigma_j] + \sum_{i=1}^N \left( \frac{(s_i - \min[s_i, \sigma_i])(\sigma_j - \min[s_j, \sigma_j])}{1 - \sum_i \min[s_i, \sigma_i]} \right)^2 \right) \right) \\ &= \frac{1}{\varepsilon} \sum_{j=1}^N s_j \max[s_j, \sigma_j] + \frac{1}{\varepsilon} \sum_{j=1}^N \sigma_j (\sigma_j - s_j) + \sum_{j=1}^N \sum_{i=1}^N \left( \frac{(s_i - \min[s_i, \sigma_i])(\sigma_j - \min[s_j, \sigma_j])}{1 - \sum_i \min[s_i, \sigma_i]} \right)^2 \end{aligned}$$

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