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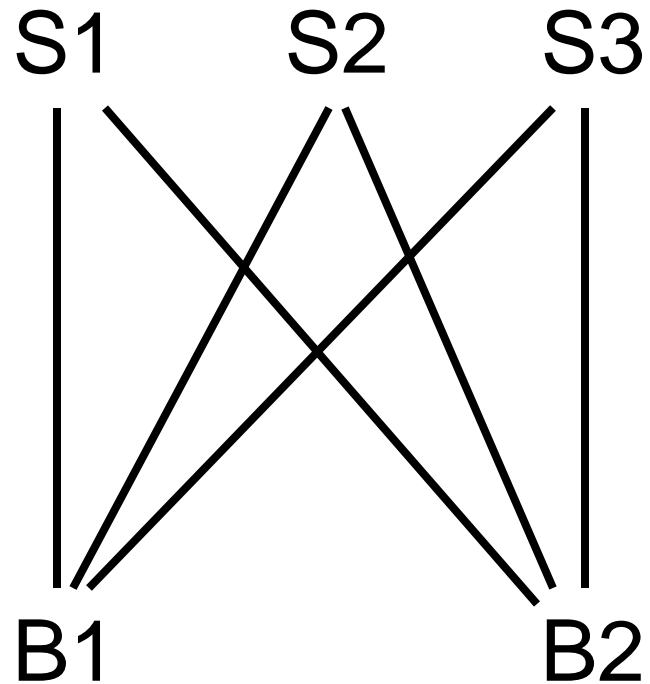
# Bilateral Bargaining with Externalities

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# Buyer-seller networks

- Imperfect competition upstream **and** downstream
- Heterogeneous firms
- Antitrust authorities and empirical IO researchers need predictions of
  - Prices and quantities
  - Incentives for Vertical and Horizontal Integration
  - Incentives for General and Specific Investments (including incentives to form “links”)
  - Entry & Exit



# Problem

= current state of multi-agent bargaining theory

- Two-person bargaining:
    - Protocol (timing of offers and counter-offers), etc. strongly affects the outcomes
  
  - Multi-agent bargaining
    - Protocol strongly affects the outcomes
    - Timing of interaction with other agents strongly affects the outcomes:
      - Do I have to break off dealing with this agent, and go back to search? What is the matching technology?
      - Can I entertain other offers, while still dealing with this agent?
      - Do all agents on one side of the market make offers at once?
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# → Two responses of IO theory

## (1) Possibility results

- Model designed to highlight a new effect that could be driving results
- Bargaining model: plausible extension of two-person bargaining to one or two more players, or of principal-agent theory, or random assignment of tiolios
- “corner solutions”

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## → Two responses of IO theory

### (2) Borrow from Cooperative game theory

- “Biform games”:
    - Bargaining stage is a black box: assume that the payoff agents receive is their payoff from a cooperative bargaining game
    - Investment stage, prior to bargaining: agents take non-cooperative actions based on how it will affect their bargaining payoff
  - Hart and Moore (1990): Shapley value
  - Brandenburger and Stuart (2006), Kranton and Minehart (2001), MacDonald and Ryall (2004): Payoffs within the Core
  - Analogy with two-person bargaining: we use Nash bargaining solution, once foundations are provided that indicate when it’s reasonable.
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# Biggest problem with cooperative bargaining

- = Cooperative bargaining assumes agents agree on the actions that **maximise the joint surplus** (the contractible actions that are bargained over)
- But what if agents can impose **negative externalities** on each other? (for instance, firms competing in the same markets)
  - What if agents **lack information** about the contracts signed by other agents: even observing a contract does not preclude additional “secret” contracts being written
  - What if they can't all negotiate at the same time, by law? Absent regular illegal meetings (or other coordinating mechanisms) buyer-seller networks have difficulty behaving like a monopolist...
- We can **assume** something other than surplus-maximising, and modify the cooperative bargaining outcome, by assumption: but what guides our assumption?

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# Non-cooperative game: Pairwise bargaining with externalities

We propose a non-cooperative bargaining model for environments in which

1. agents bargain **in pairs**, in some random sequence;
2. a pair of agents may impose **negative externalities** on other agents;
3. contract details in other pairs are **not observable**.

# Pairwise bargaining with externalities: Results

## 1. Non-cooperative surplus

- In the presence of externalities, actions are not efficient.
- Actions are **bilaterally efficient**—that is, they maximise the joint payoff of the pair, taking as given the actions of others.

## 2. Cooperative division of the surplus

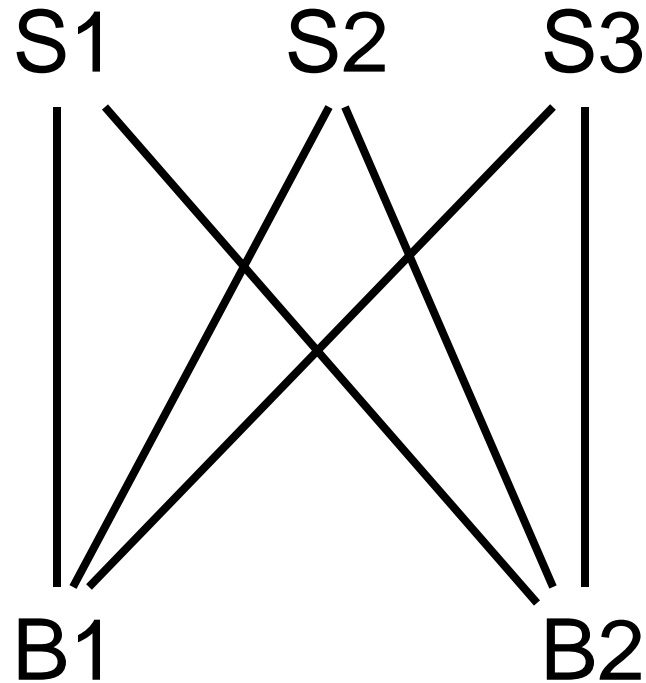
- The **reduced** surplus generated by such actions is divided according to a cooperative game theory concept—a **generalisation of Shapley values**

= basically a Shapley value extended to games with externalities and network structures

- = payoffs that are a weighted sum of profits in different industry configurations → amenable to empirical tests

# Our main application = buyer-seller networks

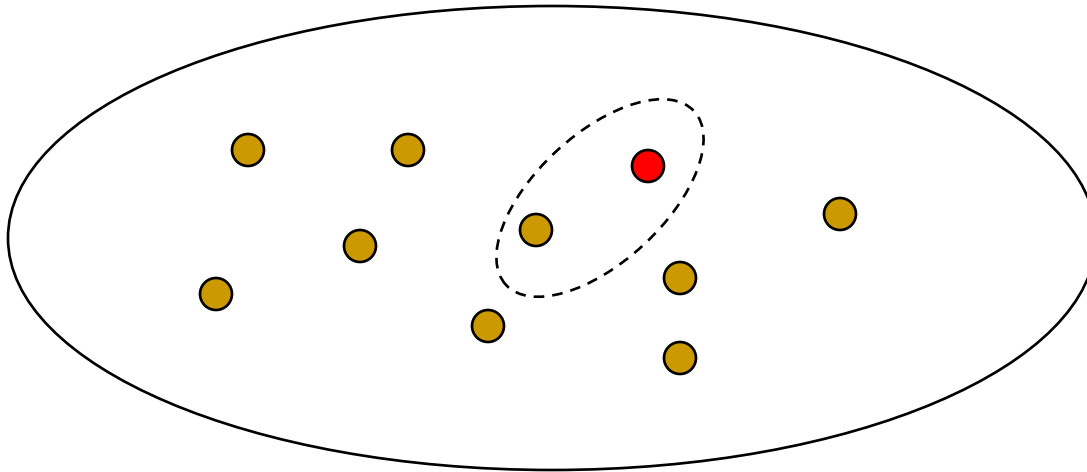
- Competitive Externalities
    - Ss may be competing in input markets
    - Bs may be competing in the same downstream market
    - can't negotiate horizontally, just negotiate supply contracts
  - Bilateral Contracts:
    - quantity and transfer payments negotiated
    - Bs and Ss cannot observe supply terms of others
  - Bilateral Efficiency:
    - **Cournot oligopoly**, if the downstream firms are in the same market (can be differentiated goods) and upstream don't compete for inputs.
- Quantities are a function of the number of downstream firms, and upstream technology.



## Related literature

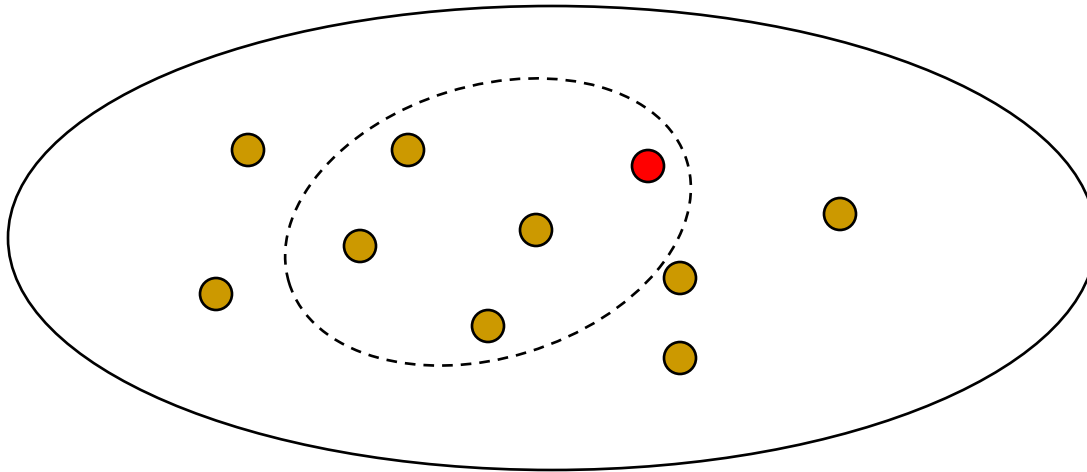
- The game we set up is a generalisation of a game by Stole and Zwiebel (1996):
  - a manager bargains with each of her workers, in turn;
  - no action space (they negotiate a wage, in exchange for a unit of labour)
  - no externalities
  - the payoff to each agent is her Shapley value.
- Their model has been extended to negotiations between upstream and downstream firms:
  - Inderst and Wey (2003)
  - Bjornerstedt and Stennek (2002);  
In both these papers, the downstream firms are in separate markets → no externalities.  
→ outcome is a Shapley value.
  - de Fontenay and Gans (2005)

# Review of cooperative game theory: Shapley values and characteristic functions



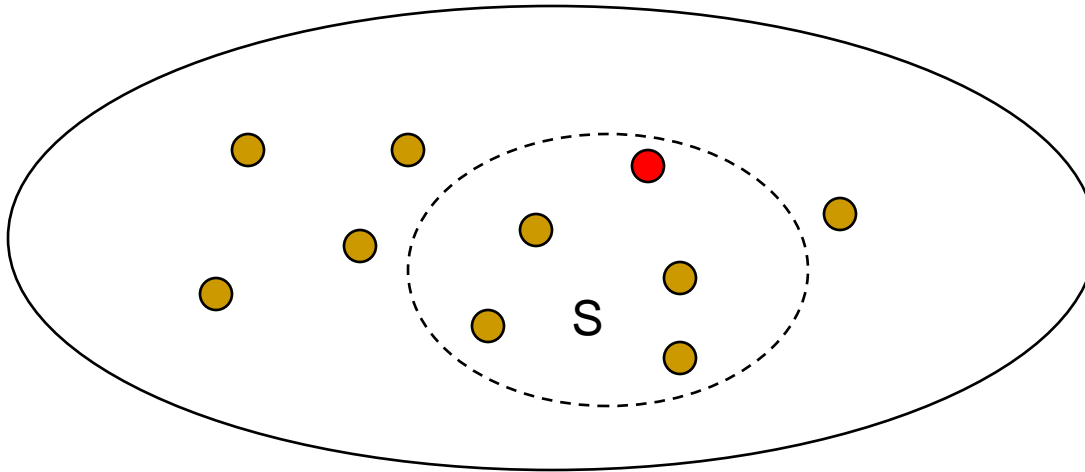
- Shapley value = a weighted sum of the **payoff** to different coalitions that you could be part of, minus their payoff without you

# Review of cooperative game theory: Shapley values and characteristic functions



- Shapley value = a weighted sum of the **payoff** to different coalitions that you could be part of, minus their payoff without you

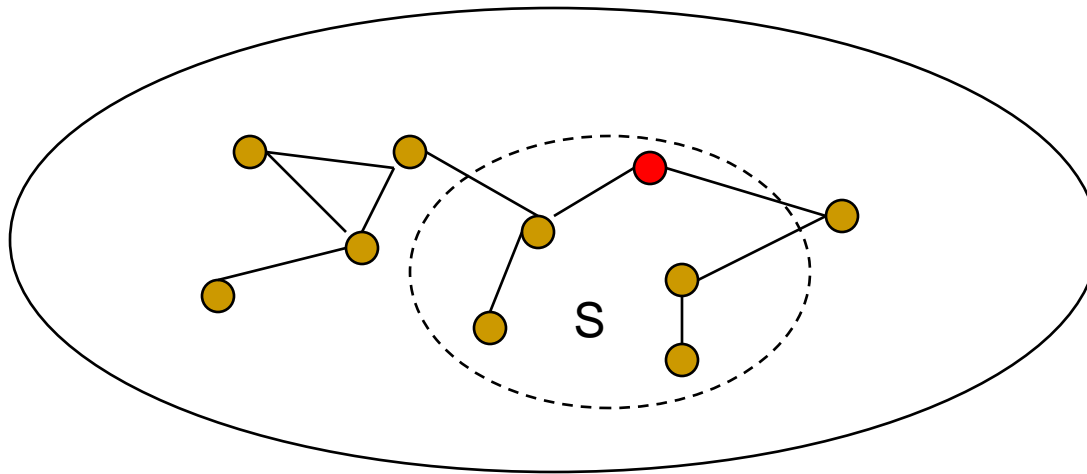
# Review of cooperative game theory: Shapley values and characteristic functions



- Shapley value = a weighted sum of the **payoff** to different coalitions that you are part of, minus their payoff without you
- The **Characteristic Function** assigns a payoff  $V(S)$  to each set  $S$ 
  - It's usually assumed that  $V(S)$  is the maximum amount that the players in  $S$  can produce together
  - $V(S)$  could be chosen in some other way; but how do you pick a likely payoff for  $V(S)$ ?

# Myerson (1977a), Jackson & Wolinsky (1996)

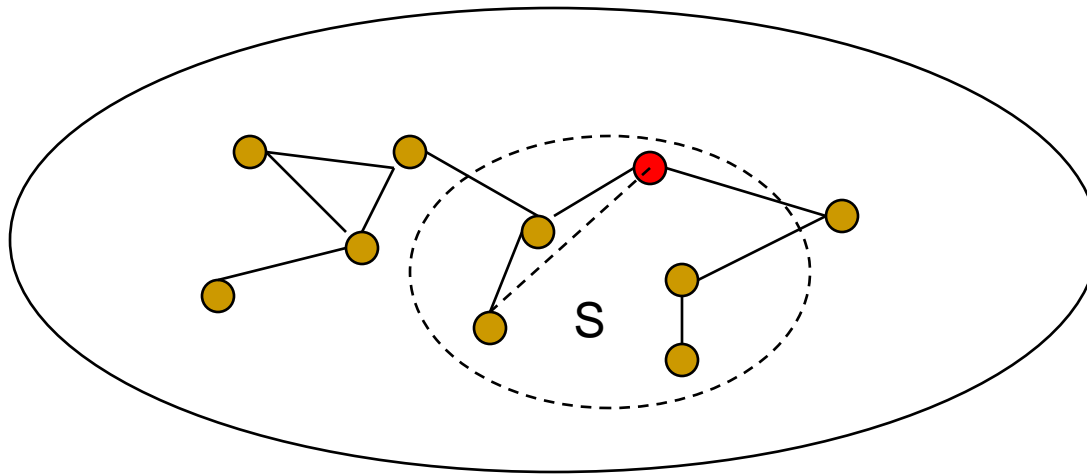
## Cooperation and Graph Structures



- Myerson (1977a): Players may only be able to cooperate with players they are linked to, in a coalition.
  - ➔ the set “S” above would become two separate coalitions, and  $V(S)$  is the sum of the payoff to each coalition
- Jackson and Wolinsky (1996): Players may not be able to cooperate as well when they are indirectly linked rather than directly linked
  - ➔ Sub-coalition S1 may produce a different amount than if it were linked by a complete graph.

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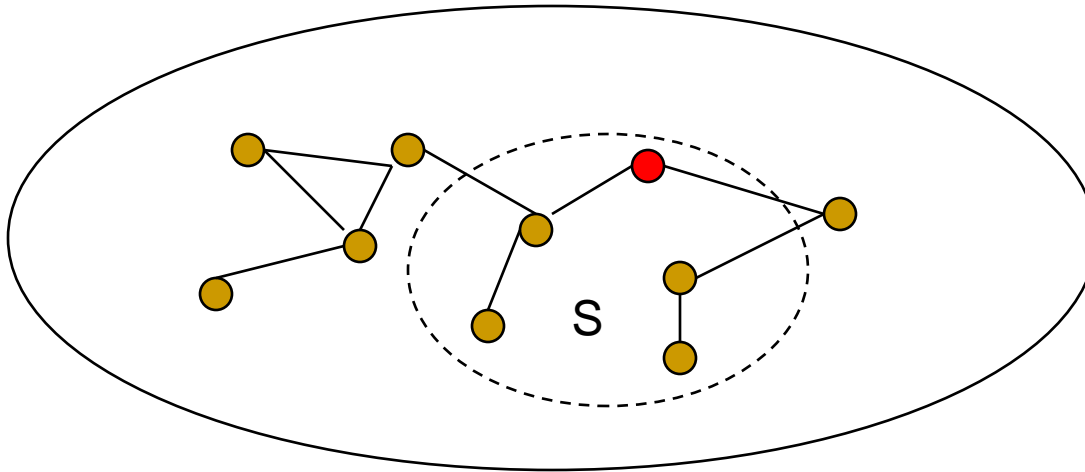
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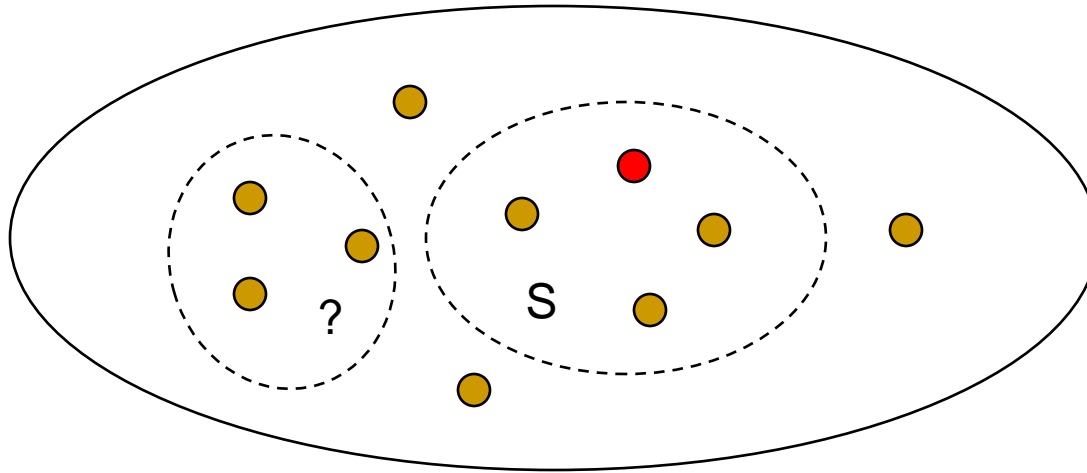
## Cooperation and Graph Structures



- The **characteristic function** becomes  $V(S, L/S)$  (where  $L/S$  is the link structure  $L$  “sliced up” by the boundaries of  $S$ )
- $V(S, L/S)$  is the sum of the payoff to each component in  $S$

$$\Phi_i(N, L) = \sum_{S \subseteq N: i \notin S} \frac{|S|!(|N|-1-|S|)!}{|N|!} (V(S \cup i, L/(S \cup i)) - V(S, L/S))$$

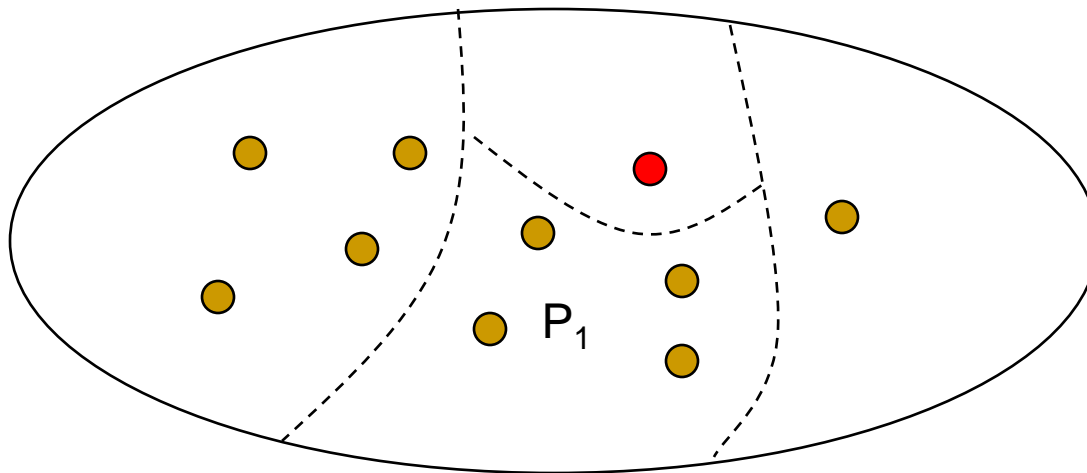
# Limitations of the Shapley value, continued



- Suppose that agents outside your coalition inflict negative externalities on you
- If it matters whether the “outsiders” are in coalitions with each other, regular Shapley values cannot accommodate that nuance.

Myerson (1977b):

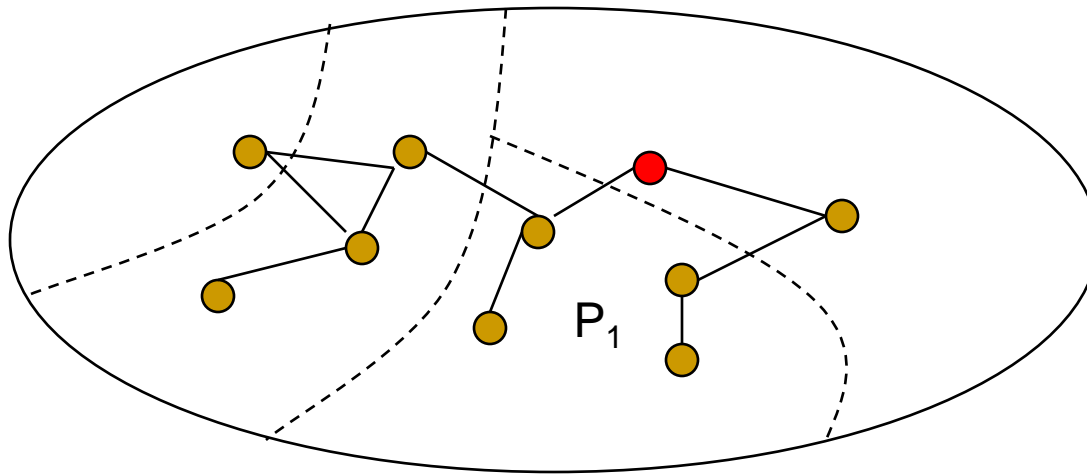
## Games in partition function form



- a characteristic function now assigns a payoff  $V(P_1, P)$  to each subset  $P_1$  of a partition  $P$  of the space.
- Remaining question: what is  $V(P_1, P)$ ?
- Myerson (1977c) envisaged each coalition maximizing its surplus, and playing a Nash equilibrium against the other coalitions: but we could choose a different  $V(P_1, P)$ .

# Navarro (2007): Graph and Partitions

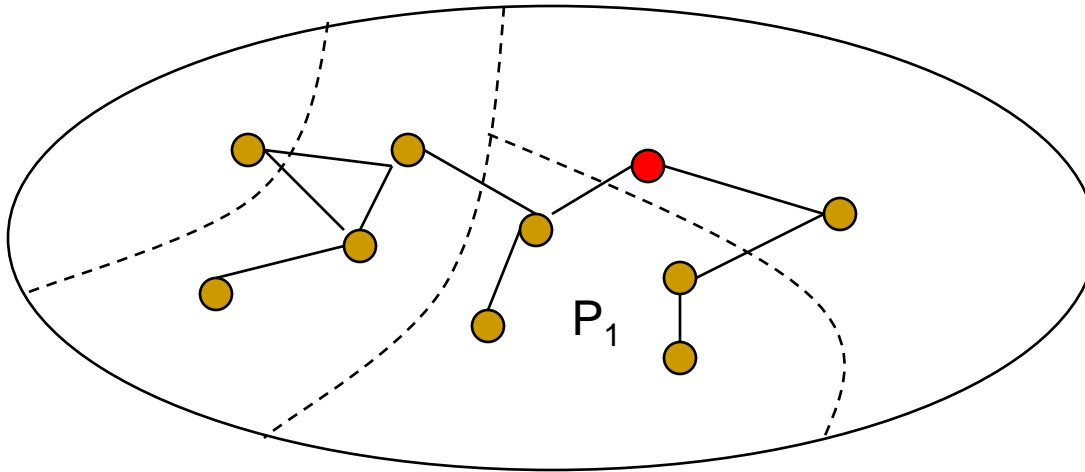
If we include a graph structure...



- Each possible partition is broken up by the graph ( $L/P$ ), and the graph structure is broken up by the partition structure ( $L^P$ ).
- Each component may potentially inflict externalities on other components
- Each component may be restricted by the graph in its ability to cooperate.

$$V(P_1, L^P) \equiv \sum_{\substack{S \in P/L \\ S \subset P_1}} V(S, L^P)$$

# Navarro (2007): Graph and Partitions



- Now there is a graph structure, further broken up by the partition structure:  $L^P$

$$\Phi_i(N, L) = \sum_{P \in \mathcal{P}^N} \sum_{S \in P} (-1)^{p-1} (p-1)! \left[ \frac{1}{|N|} - \sum_{\substack{i \notin S' \in P \\ S' \neq S}} \frac{1}{(p-1)(|N| - |S'|)} \right] V(S, L^P)$$

Call this the “generalized Shapley value”

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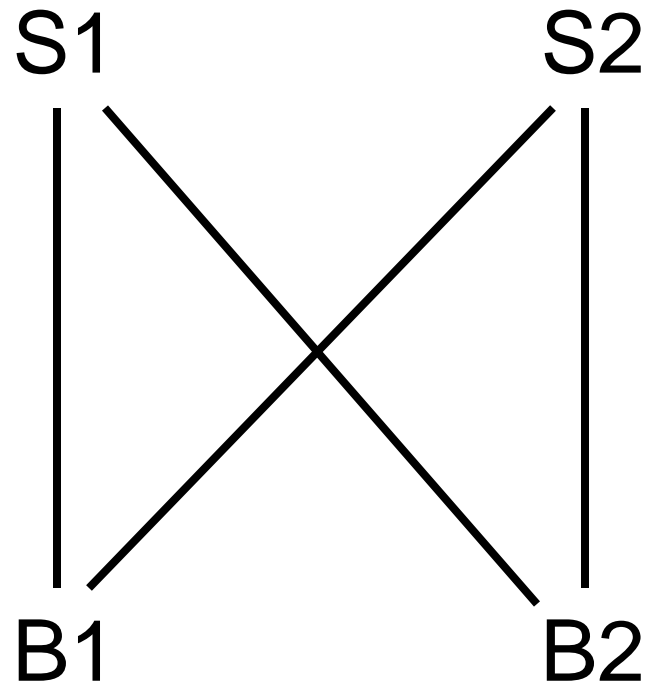
# Our non-cooperative game

The payoffs in our non-cooperative game will be generalized Shapley values, but with a specific characteristic function  $\hat{V}(S, L^P)$  derived **directly from the non-cooperative game**.

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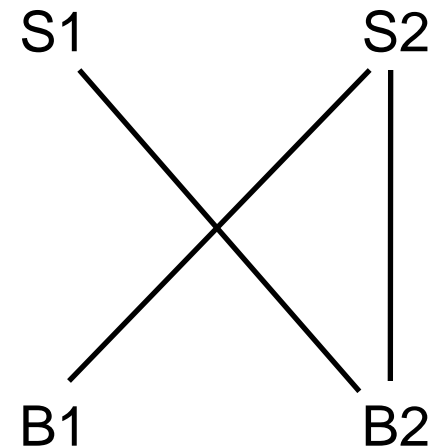
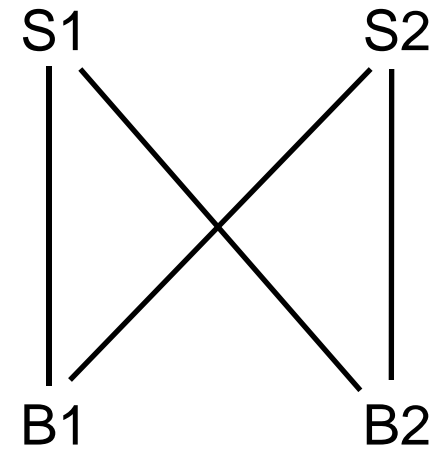
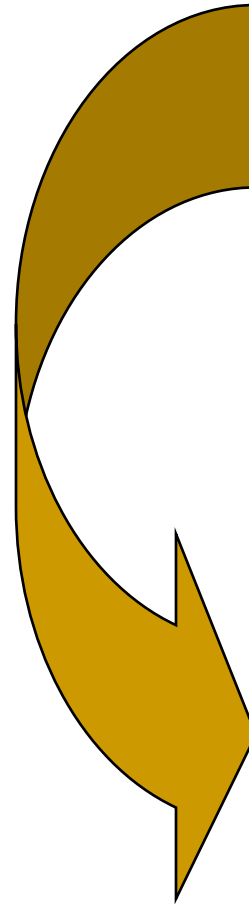
# Some Notation

- Actions
  - $x_{ij}$  is the input quantity purchased by  $B_i$  from  $S_j$
  - $t_{ij}$  is the transfer from  $B_i$  to  $S_j$
- **(A1)** You can only observe actions and transfers that you are a party to  
(e.g.,  $S_2$  and  $B_2$  cannot observe  $x_{11}$  or  $t_{11}$ )



# Network State

- Bilateral links form a graph of relationships denoted by  $K$ 
  - Initial state:  
 $K = (11,12,21,22)$   
(order = S1B1,S1B2,...)
- If a pair suffer a breakdown in negotiations (e.g., B1 and S1), a new network is created
  - New state:  $K = (12,21,22)$
- **(A2)** The network state ( $K$ ) is publicly observed



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# The Space of Possible Contracts

- **Bilaterality**
    - As terms within other pairs are unobserved by at least one member of a pair, supply terms cannot be made contingent upon other supply contract terms
  - **Network Observability**
    - If a link between any pair is broken, the other players observe it.
  - **Renegotiation**
    - After a link is broken, **any other contract** can be **unilaterally** re-negotiated, citing a “material change in circumstances” clause.
    - If a link were broken, all contracts would be re-negotiated.
-

# Extensive Form

- There exists an initial state of the network,  $K$
  - Fix an order of negotiations (in this case 4)
    - Precise order will not matter in equilibrium (under our belief structure)
  
  - Each pair negotiates in turn
    - Randomly select  $B_i$  or  $S_j$
    - That agent, say  $B_i$ , makes an offer  $\{x_{ij}(K), t_{ij}(K)\}$  including  $B_i$  and  $S_j$ .
      - If  $S_j$  accepts, the offer is fixed and move to next pair
      - If  $S_j$  rejects,
        - With probability,  $1-\sigma$ , negotiations end and bargaining recommences over the new network  $K-ij$ .
        - Otherwise negotiations continue with  $S_j$  making an offer to  $B_i$ .
- = **Binmore-Rubinstein-Wolinsky (1986) bilateral bargaining** embedded in a sequence of interrelated negotiations
- Examine outcomes as  $\sigma$  goes to 1.

# Beliefs

- Game of imperfect information
  - Need to impose some structure on out of equilibrium beliefs
  - Common issue in vertical contracting (McAfee and Schwartz; Segal)
  
- Simplifying approach: (A3) impose *passive beliefs*
  - Let  $\{\hat{x}_{ij}(K), \hat{t}_{ij}(K)\}_{\forall ij, K}$  be the set of equilibrium agreements
  - When  $i$  receives an offer from  $j$  of  $x_{ij}(K) \neq \hat{x}_{ij}(K)$  or  $t_{ij}(K) \neq \hat{t}_{ij}(K)$   $i$  does not revise its beliefs about any other outcome of the game

# Equilibrium Outcomes: Actions

- Bilateral Efficiency

A set of actions  $x_{ij}$  satisfies bilateral efficiency if for all  $ij$  in  $K$ ,

$$\hat{x}_{ij}(K) \in \arg \max_{x_{ij}} b\left(x_{ij} + x_{i,-j}; \hat{x}_{-i1}(K) + \hat{x}_{-i2}(K)\right) - c\left(x_{ij} + \hat{x}_{-i,j}(K)\right)$$

## Theorem 1:

Suppose that all agents hold passive beliefs. Then, as  $\sigma$  approaches 1, in any Perfect Bayesian equilibrium, each  $x_{ij}(K)$  is bilaterally efficient (given the state of the network  $K$ ).

# Equilibrium Outcomes: Actions

## Theorem 1:

Suppose that all agents hold passive beliefs. Then, as  $\sigma$  approaches 1, in any Perfect Bayesian equilibrium, each  $x_{ij}(K)$  is bilaterally efficient (given the state of the network,  $K$ ).

In general terms:

- Each pair of negotiators chooses the jointly efficient actions for **actions they can both observe**,  $x_{ij}$ .
- For **actions that only one player  $i$  can observe**,  $x_i$ , she takes the privately efficient action (taking other actions as given).
- Some concavity assumptions required for uniqueness.

# Equilibrium Payoffs in General Terms

## Theorem 2:

Agents receive their generalized Shapley value, where the characteristic function  $\hat{V}(S, L^P)$  for any set  $S$  in the partition is the total payoff when agents take bilaterally efficient actions with agents they're linked to.

$$\Phi_i(N, L) = \sum_{P \in P^N} \sum_{S \in P} (-1)^{p-1} (p-1)! \left[ \frac{1}{|N|} - \sum_{\substack{i \notin S' \in P \\ S' \neq S}} \frac{1}{(p-1)(|N| - |S'|)} \right] \hat{V}(S, L^P)$$

where:

- $N$  is the set of agents
- $P$  is a partition over the set of agents with cardinality  $p$
- $P^N$  is the set of all partitions of  $N$
- $L$  is the initial network (i.e., initial set of bilateral links)
- $L^P$  is the initial network with links severed between partitions defined by  $P$ .

# What drives these results?

- **If there are only two players:** Binmore-Rubinstein-Wolinsky (1986) bargaining gives the Nash bargaining outcome
  - Players agree on the surplus-maximizing actions, because they can use the lump-sum transfer to allocate profits
  - Players split the surplus = the value created by agreement, over and above their outside options:

$$\boxed{\text{Payoff to } i \text{ if } i \text{ and } j \text{ agree}} - \boxed{\text{Payoff to } i \text{ if } i \text{ and } j \text{ break off forever}} = \boxed{\text{Payoff to } j \text{ if } i \text{ and } j \text{ agree}} - \boxed{\text{Payoff to } j \text{ if } i \text{ and } j \text{ break off forever}}$$

- If two players are negotiating within a larger set of agreements, **but with passive beliefs**, they take as given:
  - what other agreements will be signed
  - what agreements will be signed in the event of a breakdown
- ➔ They agree on the surplus maximizing actions, holding others' actions constant
- ➔ The above equation holds

# What drives these results? (Stole & Zwiebel 1996)

$$\boxed{\text{Payoff to } i \text{ if } i \text{ and } j \text{ agree}} - \boxed{\text{Payoff to } i \text{ if } i \text{ and } j \text{ break off forever}} = \boxed{\text{Payoff to } j \text{ if } i \text{ and } j \text{ agree}} - \boxed{\text{Payoff to } j \text{ if } i \text{ and } j \text{ break off forever}}$$

- SZ noted that these conditions resemble Myerson's (1977) **fair allocation** conditions:

Myerson showed that if, for any graph structure  $L$  and players  $i$  and  $j$  linked together in  $L$ , the allocation to each players  $i$  and  $j$  (denoted  $\Phi_i(N, L)$  and  $\Phi_j(N, L)$ ) satisfies:

$$\Phi_i(N, L) - \Phi_i(N, L \setminus \{ij\}) = \Phi_j(N, L) - \Phi_j(N, L \setminus \{ij\})$$

that allocation is a Shapley value of the game restricted to graph structure  $L$ .

→ Myerson's result generalizes to games with externalities

# Passive beliefs: Discussion

- Passive beliefs is an unattractive assumption:
  - If an agent has been in a prior negotiation, her trading partner does not reason backwards when he sees an unexpected offer
  - Signalling equilibria are ruled out:
    - she might signal that she'd done badly in an earlier negotiation, by asking for more & refusing offers for a certain number of periods.
- It's a trembling-hand-perfect equilibrium (but not the **only** trembling-hand-perfect equilibrium).
- Alternative setups:
  - simultaneous game, with delegated agents who cannot communicate: doesn't eliminate the need for passive beliefs...
  - order uncertainty: it may be too costly to signal information about order.

## Example:

### Application to Buyer-Seller networks (de F & Gans 2005)

- Two upstream assets, two downstream assets
- Each asset is owned by its manager.

$$\Phi_{B1} = \frac{1}{12} \left( \begin{array}{l} 3\Pi^C(\overline{S_1 S_2 B_1 B_2}) \\ + \Pi^C(\overline{S_1 B_1 B_2}) + \Pi(\overline{S_1 S_2 B_1}) + \Pi^C(\overline{S_2 B_1 B_2}) - 3\Pi(\overline{S_1 S_2 B_2}) \\ - 2\Pi(\overline{S_1 B_1}) - 2\Pi(\overline{S_2 B_1}) + 2\Pi(\overline{S_1 B_2}) + 2\Pi(\overline{S_2 B_2}) \\ + 3\Pi^C(\overline{S_1 B_1} | \overline{S_2 B_2}) - 3\Pi^C(\overline{S_2 B_2} | \overline{S_1 B_1}) \\ + 3\Pi^C(\overline{S_2 B_1} | \overline{S_1 B_2}) - 3\Pi^C(\overline{S_1 B_2} | \overline{S_2 B_1}) \end{array} \right)$$

- In the absence of externalities (i.e. if B1 and B2 were in separate markets), this payoff collapses to the Shapley value.

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# Conclusion

This paper provides a game theory foundation for

- ❑ Adopting a cooperative bargaining concept: the generalized Shapley value
  - ❑ But over a reduced surplus, as defined by bilateral efficiency
  - ❑ And over a network defined by technology, location, history, or legislation
-

# Application: Vertical Integration

## Theories related to bargaining power:

- **Supply assurance**: ensure that your firm has an assured supply of inputs from upstream  
= reduce the bargaining power of upstream suppliers, by reducing the threat of non-supply from upstream
- **Vertical foreclosure**, or “Raising Rivals’ Costs”: vertical integration to worsen your competitor’s position in the market (input prices she pays, or, if you are upstream, prices she receives)
- **Property rights theory** (Grossman-Hart 1986, Hart-Moore 1990): ownership is the right to exclude other individuals from the use of assets → may raise your payoff from bargaining → may increase your incentive to invest.
- **Transactions cost theory** (Williamson 1985): agents may under-invest in assets if they do not expect to recoup the investment in bargaining → if vertical integration (ownership of upstream and downstream assets) increases the bargaining payoff to the owner of the assets, the owner will invest more.

# Application: Vertical Integration

## Theories related to the price-quantity contracting space:

- **Restricted contract space: Double marginalization.** Only price can be contracted upon → vertical integration in a one upstream – one downstream firm reduces the markup
- **Leveraging market power** (Rey and Tirole 1996 “Vertical Foreclosure”) An upstream monopolist may not be able to commit not to offer extra quantities to a competitor at a discount
  - under passive beliefs, the outcome is Cournot quantities
  - buying a downstream firm will cause the monopolist to internalize the externality she imposes on that firm = reduce downstream quantities, leverage more market power.
- **Expanded contract space: ex: Marx & Shaffer.** Players can offer nonlinear prices, breakup fees, market-share discounts, etc... (Actions are contingent on agreements with other agents.)

- Sum of the payoffs to B1 and S1, under no integration:

$$w_{B1} + w_{S1} = \frac{1}{12} \left( \begin{array}{l} 6\Pi^C(\overline{S_1 S_2 B_1 B_2}) \\ + 2\Pi^C(\overline{S_1 B_1 B_2}) + 2\Pi(\overline{S_1 S_2 B_1}) - 2\Pi^C(\overline{S_2 B_1 B_2}) - 2\Pi(\overline{S_1 S_2 B_2}) \\ - 4\Pi(\overline{S_1 B_1}) + 4\Pi(\overline{S_2 B_2}) \\ + 6\Pi^C(\overline{S_1 B_1} | \overline{S_2 B_2}) - 6\Pi^C(\overline{S_2 B_2} | \overline{S_1 B_1}) \end{array} \right)$$

- Under forward integration:

- Asymmetric Cournot;
- $\Pi(S_2 B_1 B_2)$  becomes  $\Pi(S_2 B_2)$ , because B<sub>1</sub> cannot supply S<sub>2</sub> if she has broken down negotiations with S<sub>1</sub>.

$$w_{B1} + w_{S1} = \frac{1}{12} \left( \begin{array}{l} 6\hat{\Pi}^{AC}(\overline{S_1 S_2 B_1 B_2}) \\ + 2\hat{\Pi}^{AC}(\overline{S_1 B_1 B_2}) + 2\Pi(\overline{S_1 S_2 B_1}) - 2\Pi(\overline{S_1 S_2 B_2}) \\ - 4\Pi(\overline{S_1 B_1}) + 2\Pi(\overline{S_2 B_2}) \\ + 6\Pi^C(\overline{S_1 B_1} | \overline{S_2 B_2}) - 6\Pi^C(\overline{S_2 B_2} | \overline{S_1 B_1}) \end{array} \right)$$

# Forward Integration vs. Backward Integration

- Forward integration is more profitable than backward if

$$\Pi^C(B_1B_2S_2) \geq \Pi(B_2S_1S_2)$$

Logic =

If  $S_1$  owns the assets, and employs  $B_1$  as a manager, she can reduce  $S_2$ 's bargaining power considerably, because it is no longer possible for  $S_2$  to form a coalition with  $B_1$  but without  $S_1$ .

- It's jointly profitable for the merged firm to hand power to the segment facing a relatively strong competitor
- It's more profitable for the weaker competitors to merge  
= "Raising rivals' costs"

# “Raising Rivals’ Costs” versus Investment

- Forward integration is more profitable if

$$\Pi^C(B_1 B_2 S_2) \geq \Pi(B_2 S_1 S_2)$$

- But that will imply a big fall in  $B_1$ 's incentive to invest, as the  $\Pi^C(B_1 B_2 S_2)$  term will drop out of her payoff.
- The traditional property rights motive for integrating works against the incentive to reduce competitors' bargaining power
- Which is more important depends on the potential for investment in  $B_1$  and  $S_1$  productivity.

# Supply assurance, etc.

- Physical capital is alienable
- ➔ In the absence of **irreplaceable** human capital (or the potential for it), selling it all to one agent could solve the investment problem...

(One interpretation of the difference between the Transactions Cost (TC) literature and the Property Rights (PR) literature is that the former is about physical investments in the absence of valuable human capital.)

- In such an environment, a monopsonist buying out some of his sellers eliminates the need to bargain with them (he can replace them as managers, at will)
- ➔ Has a strong effect on bargaining power of sellers = supply assurance!
- In this situation, the “bargaining power” effect and the “investment” effect complement each other.