

Table 1: Payoffs with no externalities¹

Upstream Competition	Upstream Monopoly (U_A owns U_B)
$v_{D_1} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) + \Pi(\overline{D_1 U_A}) - 3\Pi(\overline{D_2 U_B})$ $+ x \left(\Pi(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B}) + \Pi(\overline{D_1 U_B}) \right)$ $\left. + y \left(-3[\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B})] - \Pi(\overline{D_2 U_A}) \right) \right)$	$v_{D_1} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) + \Pi(\overline{D_1 U_A})$ $+ (1-y)\Pi(\overline{D_1 D_2 U_B}) - 3y\Pi(\overline{D_2 U_A U_B})$ $\left. - y\Pi(\overline{D_2 U_A}) + (1-y)\Pi(\overline{D_1 U_B}) \right)$
$v_{D_2} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $- 3\Pi(\overline{D_1 U_A U_B}) - \Pi(\overline{D_1 U_A}) + 3\Pi(\overline{D_2 U_B})$ $+ x \left(\Pi(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B}) - \Pi(\overline{D_1 U_B}) \right)$ $\left. + y \left(\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B}) + \Pi(\overline{D_2 U_A}) \right) \right)$	$v_{D_2} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $- 3\Pi(\overline{D_1 U_A U_B}) - \Pi(\overline{D_1 U_A})$ $+ (1-y)\Pi(\overline{D_1 D_2 U_B}) + y\Pi(\overline{D_2 U_A U_B})$ $\left. + y\Pi(\overline{D_2 U_A}) - (1-y)\Pi(\overline{D_1 U_B}) \right)$
$v_{U_A} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) + \Pi(\overline{D_1 U_A}) - 3\Pi(\overline{D_2 U_B})$ $+ x \left(-3[\Pi(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B})] - \Pi(\overline{D_1 U_B}) \right)$ $\left. + y \left(\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B}) + \Pi(\overline{D_2 U_A}) \right) \right)$	$v_{U_A} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) + \Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) + \Pi(\overline{D_1 U_A})$ $- 3(1-y)\Pi(\overline{D_1 D_2 U_B}) + y\Pi(\overline{D_2 U_A U_B})$ $\left. + y\Pi(\overline{D_2 U_A}) - (1-y)\Pi(\overline{D_1 U_B}) \right)$
$v_{U_B} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) - 3\Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) - \Pi(\overline{D_1 U_A}) + 3\Pi(\overline{D_2 U_B})$ $+ x \left(\Pi(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B}) + \Pi(\overline{D_1 U_B}) \right)$ $\left. + y \left(\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B}) - \Pi(\overline{D_2 U_A}) \right) \right)$	$v_{U_B} = \frac{1}{12} \left(3\Pi(\overline{D_1 D_2 U_A U_B}) - 3\Pi(\overline{D_1 D_2 U_A}) \right.$ $+ \Pi(\overline{D_1 U_A U_B}) - \Pi(\overline{D_1 U_A})$ $+ (1-y)\Pi(\overline{D_1 D_2 U_B}) + y\Pi(\overline{D_2 U_A U_B})$ $\left. - y\Pi(\overline{D_2 U_A}) + (1-y)\Pi(\overline{D_1 U_B}) \right)$
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$\frac{\partial(v_{D_1} + v_{U_A})}{\partial(-x)} = \frac{1}{6} (\Pi(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B}))$	$\frac{\partial(v_{D_1} + v_{U_A})}{\partial(-x)} = 0$
$\frac{\partial(v_{D_1} + v_{U_A})}{\partial(-y)} = \frac{1}{6} (\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B}))$	$\frac{\partial(v_{D_1} + v_{U_A})}{\partial(-y)} = \frac{1}{6} (\Pi(\overline{D_2 U_A U_B}) - \Pi(\overline{D_1 D_2 U_B}))$

¹Where $(x, y) = (1, 1)$ for NI, $(0, 1)$ for FI, and $(1, 0)$ for BI.

Table 2: (Continued)

$\begin{aligned} \frac{\partial(\nu_{D_1+\nu_{U_A}})}{\partial(-x)} \Big _{y=1} &= \frac{1}{2} \left(\hat{\Pi}_{UC}(\overline{D_1 D_2 U_A U_B}) - \hat{\Pi}(\overline{D_1 D_2 U_A U_B}) \right) \\ &+ \frac{1}{6} \left(\hat{\Pi}(\overline{D_1 D_2 U_A}) - \hat{\Pi}(\overline{D_1 D_2 U_A}) \right) \\ &+ \hat{\Pi}(\overline{D_1 D_2 U_B}) - \Pi(\overline{D_2 U_B}) \end{aligned}$	$\begin{aligned} \frac{\partial(\nu_{D_1+\nu_{U_A}})}{\partial(-x)} \Big _{y=1} &= \frac{1}{2} \left(\hat{\Pi}_{UM}(\overline{D_1 D_2 U_A U_B}) - \hat{\Pi}(\overline{D_1 D_2 U_A U_B}) \right) \\ &+ \frac{1}{6} \left(\hat{\Pi}(\overline{D_1 D_2 U_A}) - \hat{\Pi}(\overline{D_1 D_2 U_A}) \right) \end{aligned}$
$\begin{aligned} \frac{\partial(\nu_{D_1+\nu_{U_A}})}{\partial(-y)} \Big _{x=1} &= \frac{1}{2} \left(\hat{\Pi}_{UC}(\overline{D_1 D_2 U_A U_B}) - \hat{\Pi}(\overline{D_1 D_2 U_A U_B}) \right) \\ &+ \frac{1}{6} \left(\hat{\Pi}(\overline{D_1 D_2 U_A}) - \hat{\Pi}(\overline{D_1 D_2 U_A}) \right) \\ &+ \hat{\Pi}(\overline{D_2 U_A U_B}) - \Pi(\overline{D_2 U_B}) \end{aligned}$	$\begin{aligned} \frac{\partial(\nu_{D_1+\nu_{U_A}})}{\partial(-y)} \Big _{x=1} &= \frac{1}{2} \left(\hat{\Pi}_{UM}(\overline{D_1 D_2 U_A U_B}) - \hat{\Pi}(\overline{D_1 D_2 U_A U_B}) \right) \\ &+ \frac{1}{6} \left(\hat{\Pi}(\overline{D_1 D_2 U_A}) - 2\hat{\Pi}(\overline{D_1 D_2 U_A}) \right) \\ &+ \Pi(\overline{D_2 U_A U_B}) \end{aligned}$
